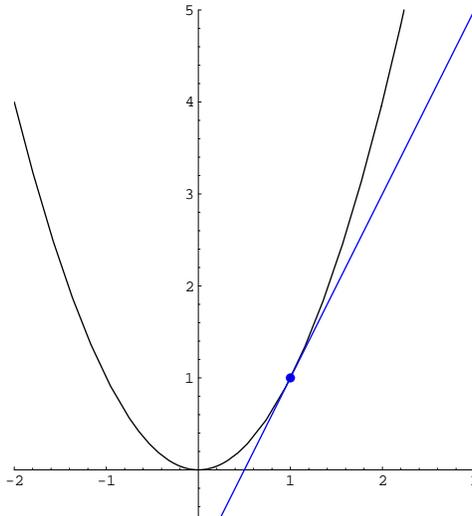**Math Xa Fall 2003**
Worksheet: The Tangent Problem
October 15, 2003

A *tangent line* to a curve is a line that touches the curve. Below is a graph of $y = x^2$ and the line tangent to it at the point $(1, 1)$.

Your task is to determine the equation of this tangent line. To do so, you will need a *point* on the line and the *slope* of the line. We know a point on the line – the point $(1, 1)$. However, we do not have the slope and we cannot find it directly. To find the slope of a line, we need *two* points on the line. We have only *one*.

Instead of finding the slope of the tangent line, we will find the slope of a line that is pretty close to the tangent line. Note that the points $(1, 1)$ and $(2, 4)$ are on the curve.

1. Draw the line that passes through the points $(1, 1)$ and $(2, 4)$ on the graph above.

We call this a *secant line* because it intersects the curve in two points. Note that the slope of the secant line is pretty close to the slope of the tangent line. Since we know two points on the secant line, we can find its slope.

2. What is the slope of the secant line to the curve $y = x^2$ through the points $(1, 1)$ and $(2, 4)$?

This secant line gives us an *approximation* of the slope of the tangent line. However, it is a pretty rough approximation, since the secant and tangent lines are close, but not that close. Instead of looking at the secant line through $(1, 1)$ and $(2, 4)$, let us find a secant line that is even closer to the tangent line. Our new secant line should still go through $(1, 1)$ because that is the only point we know on the tangent line.

3. Pick another point on the curve $y = x^2$ through which our secant line could go that would give us a better approximation to the tangent line. What point did you pick? Why did you pick this point?

4. Draw the secant line that passes through $(1, 1)$ and the point you picked in question 3 on the following graph.
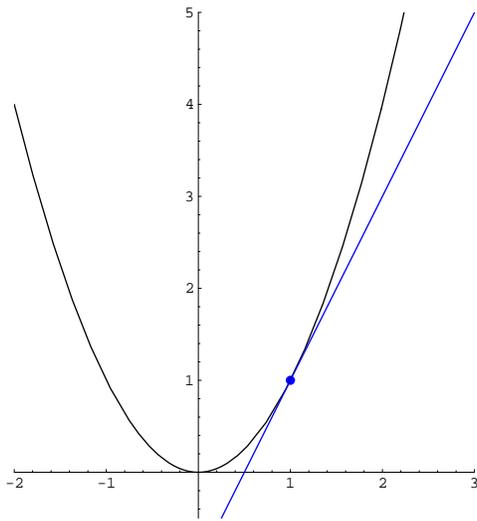
5. What is the slope of the secant line you drew in question 4?

6. Pick yet another point on the curve $y = x^2$ through which our secant line could go that would give an even better approximation to the tangent line. What point did you pick? Why did you pick this point?

7. What is the slope of the secant line through $(1, 1)$ and the point you picked in question 6?

8. Based on your calculations above, what do you think the slope of the tangent line actually is?

9. Now that we have a point on the tangent line and we think we know the slope of the tangent line, find an equation of the tangent line.