Math Xa Fall 2003
Worksheet: The Velocity Problem
October 17, 2003

Let's say you're at a swimming pool and you decide that you're going to do a belly flop off of the high dive. You're about to climb the ladder when your calculus teacher, who is coincidentally at the same pool, stops you and says, "That diving board is 32 feet above the surface of the water. Are you sure you want to do a belly flop? Do you know how fast you'll be going when you hit the water?" (Your calculus teacher cares about your education *and* your water safety.)

Let's try to find the answer to that question. How fast will you be going when you hit the water? To get us started, let's say that after t seconds, your height above the water in feet is given by the function $h(t) = -16t^2 + 32$.

1. At what height will you be when you hit the water?

2. After how many seconds will you hit the water?

To find your *instantaneous velocity* when you hit the water, we'll start by looking at your *average velocity* over a few time intervals. Your average velocity over a given time interval is your change in position divided by the change in time.

3. What is your average velocity over the entire dive? (That is, what is your average velocity from time $t = 0$ to the time value you found in question 2.)

The velocity you found in question 3 gives you a rough estimate of your instantaneous velocity at the end of your dive. To get a better estimate of your final velocity, let's look at your average velocity over increasingly smaller time intervals.

4. What is your average velocity from time t to the time value you found in question 2? (Express your answer in terms of t. Do not simplify.)

5. Substitute 0 for t in the formula you found in question 4. Do you get the same average velocity you found in question 3?

To find better estimates of your final velocity, find your average velocity over increasingly smaller time

intervals.

6. Pick three t-values between 0 and the impact time you found in question 2. Make sure that the t-values you pick get closer and closer to the impact time.

7. Write down a table with the t-values you chose in question 6 in one column and the corresponding average velocities (using your formula from question 4) in the other.

8. Based on your data from question 7, what do you think your impact velocity is?

Your instantaneous velocity at impact time is the *limit* of your average velocities between time t and your impact time as t approaches your impact time.

9. Using the formula you found in question 4, express your impact velocity as a limit. Then evaluate this limit to find your actual impact velocity. Was your guess in question 8 accurate?

10. The limit you evaluated in question 9 gave you a velocity in feet per second. What is this velocity in miles per hour? Would you want to do a belly flop knowing what you know now?