Problem Set 11 10/22/04
4.4 # 3, 4, 10
5.1 # 1

Problem 3.

(a) Equation of the supply curve:
the slope m is given by $m = \frac{12}{9} = \frac{4}{3}$. Equation of the line is $y = \frac{4}{3}(x-9)$ or $y = \frac{4}{3}x - 12$
Equation of the demand curve:
The slope m is given by $m = \frac{0-16}{12} = -\frac{4}{3}$

(b) 6.75
Equation for the line becomes: $y = -\frac{4}{3}(x-12)$ or $y = -\frac{4}{3}x + 16$.

Problem 4.

$$C(t) = \begin{cases} 250 & ,0 < t \leq 2 \\ 250 + 100(t-2) & ,t > 2 \end{cases}.$$

Problem 10.

Let $M(t)$ be the total daily mileage after t hours. The two points $(0,3)$ and $(\frac{1}{3}, \frac{13}{3})$ are the graph of $M(t)$. As $M(t)$ is linear, we calculate its slope to be $m = \frac{\frac{13}{3}-3}{\frac{1}{3}-0} = 4$ and use the slope-intercept form to obtain the equation $M(t) = 3 + 4t$. They stop when $M(t) = 13$, which occurs at $t = 2.5$ hours. The domain of $M(t)$ is $[0, 2.5]$.

Problem 1.

(a) By completing the square, we see that $h(t) = -16t^2 + 8t + 48 = -16\left(t^2 - \frac{1}{2}t\right) + 48 = -16\left(t - \frac{1}{4}\right)^2 + 49$. The graph of h is a parabola that opens downward with its vertex at $\left(\frac{1}{4}, 49\right)$. Therefore, since $1 > \frac{1}{4}$, the ball is heading down at $t = 1$.

(b) The average rate of change of height with respect to time over $[0.9, 1] = \frac{\Delta h}{\Delta t} = \frac{h(1)-h(0.9)}{1-0.9} = \frac{-16(1)^2+8(1)+48-(-16(0.9)^2+8(0.9)+48)}{0.1} = -22.4$ ft/sec.
The average rate of change of height with respect to time over $[1, 1.1] = \frac{\Delta h}{\Delta t} = \frac{h(1.1)-h(1)}{1.1-1} = \frac{-16(1.1)^2+8(1.1)+48-(-16(1)^2+8(1)+48)}{0.1} = -25.6$ ft/sec.

The ball's velocity is between -25.6 ft/sec and -22.4 ft/sec.

(c) The average rate of change of height with respect to time over $[0.99, 1] = \frac{\Delta h}{\Delta t} = \frac{h(1)-h(0.99)}{1-0.99} = \frac{-16(1)^2+8(1)+48-(-16(0.99)^2+8(0.99)+48)}{0.01} = -23.84$ ft/sec.
The average rate of change of height with respect to time over $[1, 1.01] = \frac{\Delta h}{\Delta t} = \frac{h(1.01)-h(1)}{1.01-1} = \frac{-16(1.01)^2+8(1.01)+48-(-16(1)^2+8(1)+48)}{0.01} = -24.16$ ft/sec.

The ball's velocity is between -24.16 ft/sec and -23.84 ft/sec.

(d) $h'(1) = \lim_{\Delta t \to 0} \frac{h(1+\Delta t)-h(1)}{\Delta t} = \lim_{\Delta t \to 0} \frac{-16(1+\Delta t)^2+8(1+\Delta t)+48-(-16(1)^2+8(1)+48)}{\Delta t} = \lim_{\Delta t \to 0} \frac{-32\Delta t - 16(\Delta t)^2 + 8\Delta t}{\Delta t} = \lim_{\Delta t \to 0} -16\Delta t - 24 = -24$. The instantaneous velocity of the ball at $t = 1$ is -24 ft/sec.