

- (a) The slope of the secant line through  $P$  and  $Q = m(h) = \frac{f(1+h)-f(1)}{h} = \frac{(1+h)^3-1^3}{h} = \frac{h^3+3h^2+3h}{h} = h^2 + 3h + 3$ . Now  $m(-0.1) = 2.71$ ,  $m(-0.01) = 2.9701$ ,  $m(-0.001) = 2.99700$ ,  $m(0.0001) = 3.00030$ ,  $m(0.001) = 3.00300$ ,  $m(0.01) = 3.0301$ , and  $m(0.1) = 3.31$ .
- (b)  $f'(1) = \lim_{h \rightarrow 0} m(h) = \lim_{h \rightarrow 0} (h^2 + 3h + 3) = 3$ .
- (c) The function  $f(x) = x^3$  is increasing at an increasing rate; that is, the slopes of tangent lines are nonnegative and increasing as  $x$  increases. Thus any secant line with  $h > 0$  will have a greater slope than the slope of the tangent line at  $x = 1$ , and any secant line with  $h < 0$  will have a slope less than the slope of the tangent line at  $x = 1$ . Therefore, the difference quotients for  $h > 0$  are greater than  $f'(1)$ , and the difference quotients for  $h < 0$  are less than  $f'(1)$ .

## Problem 5.

$$f'(9) = \frac{1}{6} \approx 0.16667$$

$2+h$	8.9	8.99	8.999	9.0001	9.001	9.01	9.1
$\frac{f(2+h)-f(2)}{h}$	0.16713	0.16671	0.16667	0.16667	0.16666	0.16662	0.16621

## Problem 13.

(a)  $m = \frac{f(w)-f(3)}{w-3}$

(b) (iii)

- (c) Expression i. corresponds to the slope of a secant line when  $w \neq 3$ . Expression ii. limits to the slope of the secant line to  $f$  through the points  $P = (3, f(3))$  and  $Q = (w, f(w))$ . Expression iii. is the slope of the tangent line to the graph of  $f$  at  $P$  because  $Q$  approaches  $P$  as  $w$  approaches 3. Figure 5.4 in the text illustrates this concept.

## Problem 17.

(a)  $A = (4-w, g(4-w))$ ,  $B = (4, g(4))$ ,  $C = (4+w, g(4+w))$ ,  $D = (s, g(s))$ ,  $E = (s+p, g(s+p))$ ,  
 $F = (r, g(r))$

(b) i) B, ii) C, iii) E, iv) A, v) G, vi) F, vii) D, viii) G