SECTION 6.4

THE FREE FALL OF AN APPLE

Problem 4.

(a) $p = 1000 - 4q \Leftrightarrow q = \frac{1000-p}{4} \Rightarrow R(p) = pq = p\left(\frac{1000-p}{4}\right) = -\frac{1}{4}p^2 + 250p$. As the revenue function R is quadratic, it is maximized at the value of p at which $R'(p) = 0$. Now $R'(p) = -\frac{1}{2}p + 250$, which gives us that $R'(p) = 0$ when $p = 500$. Thus revenue is maximized at a price level of \$500.

(b) The maximum revenue is $R(500) = 62,500$; to find the price levels that give half of the maximum profit, we solve $R(p) = 31,250$ for p. Now $-\frac{1}{4}p^2 + 250p - 31,250 = 0 \Leftrightarrow p^2 - 1000p + 125,000 = 0 \Rightarrow p = \frac{1000 \pm \sqrt{500,000}}{2} \Rightarrow p \approx 146.45$ or $p \approx 853.55$. Therefore, the prices that give half of the maximum revenue are \$146.45 and \$853.55.

Problem 8.

(a) The apples are thrown from heights of $B(0) = 10$ ft and $D(0) = 10$ ft.

(b) As height function $B(t)$ of Ben's apples is quadratic, the maximum height of his apples is achieved at the value of t for which $B'(t) = 0$. Now $B'(t) = -32t + 4$, and hence $B'(t) = 0$ when $t = \frac{1}{8}$. Therefore, the maximum height of Ben's apple is $B(\frac{1}{8}) = -16\left(\frac{1}{8}\right)^2 + 4\left(\frac{1}{8}\right) + 10 = 10.25$ ft.

(c) As $D'(t) = -32t - 2$, the velocity of David's apple is always negative from the time the apple is thrown, which means that his apple is falling as from the time it is thrown until its hits the ground. Therefore, the maximum height of David's apples is the initial height of $D(0) = 10$ ft.

(d) The initial velocity of Ben's apple is $B'(0) = 4$ ft/sec > 0 ft/sec, which indiates that Ben tossed his apple upward, and the initial velocity of David's apple is $D'(0) = -2$ ft/sec < 0 ft/sec, which indicated that David tossed his apple downward.

(e) Ben's apple hits the ground at the time $t > 0$ for which $0 = B(t) = -16t^2 + 4t + 10$. Now $-16t^2 + 4t + 10 = 0 \Leftrightarrow 8t^2 - 2t - 5 = 0 \Rightarrow t = \frac{2 \pm \sqrt{(-2)^2 - 4(8)(-5)}}{16} = \frac{1 \pm \sqrt{41}}{8}$. Taking the positive value for t, we have $t = \frac{1+\sqrt{41}}{8} \approx 0.925$ seconds. Therefore, Ben's apples hits the ground 0.925 seconds after being tossed.

Problem 1.

(a) $\lim_{x \to \infty} (1.1)^x = \infty$

(b) $\lim_{x \to \infty} (0.9)^x = 0$

(c) $\lim_{x \to 0} (1.1)^x = 1$

(d) $\lim_{x \to -\infty} (1.1)^x = 0$

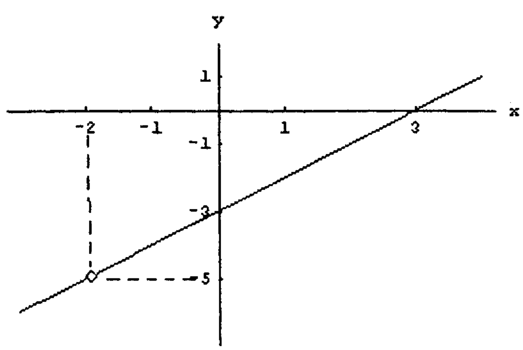
(e) $\lim_{x \to -\infty} (0.9)^x = \infty$

if $0 < b < 1$ then $\lim_{x \to \infty} b^x = 0$
if $b > 1$ then $\lim_{x \to \infty} b^x = \infty$

Problem 3.

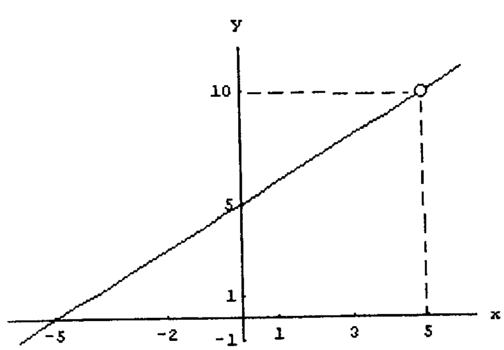
(a) $\lim_{h \to -2} \frac{(h-3)(h+2)}{(h+2)} = \lim_{h \to -2} (h-3) = -5$

$f(x) = x - 3,\ D : x \neq -2$



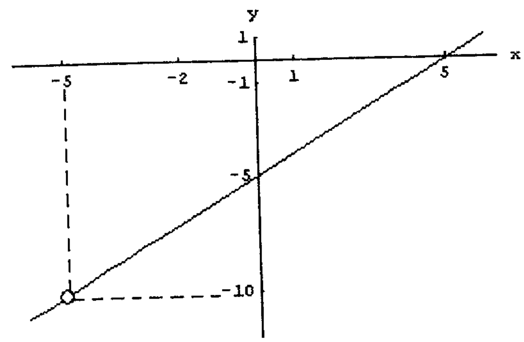
(b) $\lim_{x \to 5} \frac{x^2 - 25}{x - 5} = \lim_{x \to 5} \frac{(x-5)(x+5)}{x-5} = \lim_{x \to 5} (x+5) = 10$

$f(x) = \frac{x^2-25}{x-5} = x + 5,\ D : x \neq 5$



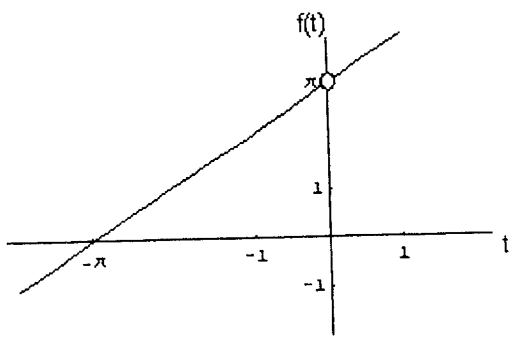
(c) $\lim_{x\to-5}\frac{x^2-25}{x+5}=\lim_{x\to-5}\frac{(x-5)(x+5)}{x+5}=\lim_{x\to-5}(x-5)=-10$

$f(x)=\frac{x^2-25}{x+5}=x-5,\ D: x\neq-5$



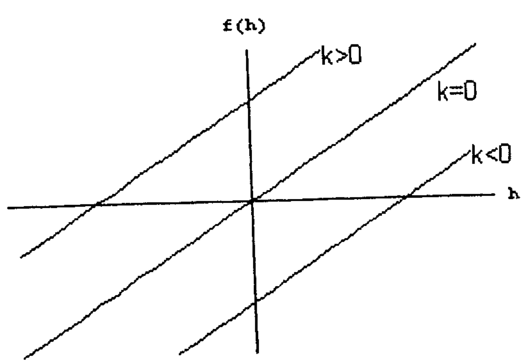
(d) $\lim_{t\to0}\frac{t^2+\pi t}{t}=\lim_{t\to0}\frac{t(t+\pi)}{t}=\lim_{t\to0}(t+\pi)=\pi$

$f(t)=\frac{t^2\pi t}{t}=t+\pi,\ D: t\neq0$



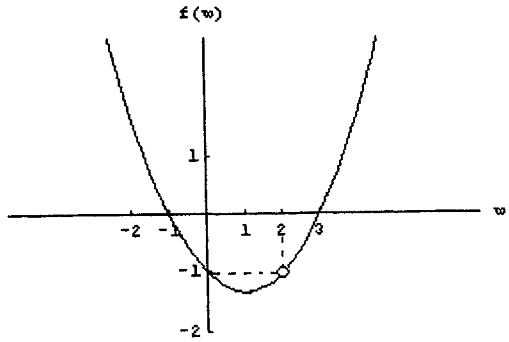
(e) $\lim_{h\to0}\frac{hk+h^2}{h}=\lim_{h\to0}\frac{h(k+h)}{h}=\lim_{h\to0}(k+h)=k$

$f(h)=\frac{hk^2+h^2}{h}=h+k$, k-constant $D: h\neq0$



(f) $\lim_{w\to 2} \frac{(w-3)(w+1)(w-2)}{3w-6} = \lim_{w\to 0} \frac{(w-3)(w+1)(w-2)}{3(w-2)} = \lim_{w\to 0} \frac{(w-3)(w+1)}{3} = \frac{-1\cdot 3}{3} = -1$

$f(w) = \frac{(w-3)(w+1)(w-2)}{3(w-2)} = \frac{1}{3}(w-3)(w+1),\ D: w \neq 2$

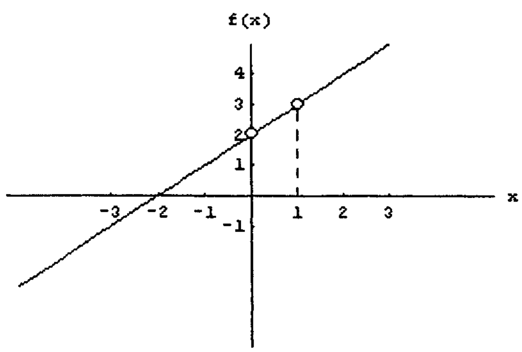


Problem 10.

$f(x) = \frac{(x+2)(x^2-x)}{x(x-1)}$

(a) $\lim_{x\to 0} \frac{(x+2)(x^2-x)}{x(x-1)} = 2$

(b) $\lim_{x\to 1} \frac{(x+2)(x^2-x)}{x(x-1)} = 3$



Problem 15.

$f(x) = \sqrt{x},\ a = 9,\ f'(9) = \lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} = \lim_{h\to 0} \frac{(\sqrt{9+h}-3)(\sqrt{9+h}+3)}{h(\sqrt{9+h}+3)} = \lim_{h\to 0} \frac{9+h-9}{h(\sqrt{9+h}+3)} =$

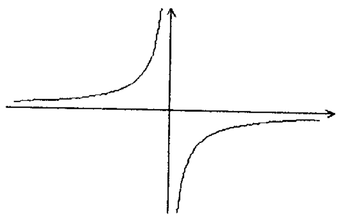
$\lim_{h\to 0} \frac{1}{\sqrt{9+h}+3} = \frac{1}{6}$

Problem 16.

$f(x) = e^x;\ a = 1;\ \lim_{h\to 0} \frac{e^{1+h}-e}{h} = f'(1) = e$

Problem 1.

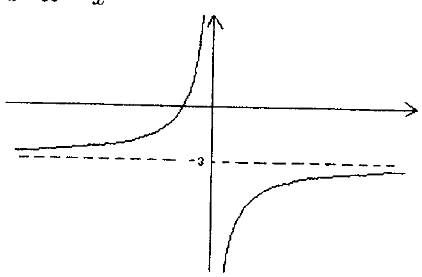
(a)



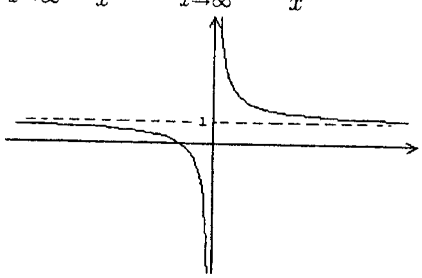
(i) $\lim_{x \to -\infty} (-\frac{3}{x}) = 0$

(ii) $\lim_{x \to \infty} (-\frac{3}{x}) = 0$

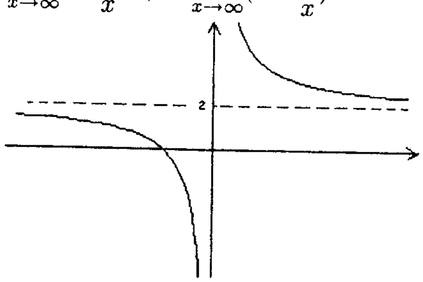
(iii) $\lim_{x \to \infty} (-\frac{3}{x} - 3) = -3$



(iv) $\lim_{x \to \infty} (\frac{x+1}{x}) = \lim_{x \to \infty} (1 + \frac{1}{x}) = 1$



(v) $\lim_{x \to \infty} (\frac{2x+3}{x}) = \lim_{x \to \infty} (2 + \frac{3}{x}) = 2$



(b) (i) $\lim_{x \to -\infty} (-\frac{3}{x}) = \lim_{x \to \infty} (\frac{3}{x}) = \lim_{x \to \infty} (3) \cdot \lim_{x \to \infty} (\frac{1}{x}) = 3 \cdot 0 = 0$

(ii) $\lim_{x \to \infty} (-\frac{3}{x}) = \lim_{x \to \infty} (-3) \cdot \lim_{x \to \infty} (\frac{1}{x}) = -3 \cdot 0 = 0$

(iii) $\lim_{x \to \infty} (-\frac{3}{x} - 3) = \lim_{x \to \infty} (-3) \cdot \lim_{x \to \infty} (\frac{1}{x}) - \lim_{x \to \infty} (3) = -3 \cdot 0 - 3 = -3$

(iv) $\lim_{x \to \infty} (\frac{x+1}{x}) = \lim_{x \to \infty} (1 + \frac{1}{x}) = \lim_{x \to \infty} (1) + \lim_{x \to \infty} (\frac{1}{x}) = 1 + 0 = 1$

(v) $\lim_{x \to \infty} (\frac{2x+3}{x}) = \lim_{x \to \infty} (2 + \frac{3}{x}) = \lim_{x \to \infty} (2) + \lim_{x \to \infty} (3) \cdot \lim_{x \to \infty} (\frac{1}{x}) = 2 + 3 \cdot 0 = 2$