Section 7.4 Continuity and the Intermediate and Extreme Value Theorems

Problem 2.

(a) ∞

(b) ∞

(c) ∞

(d) ∞

(e) $-\infty$

(f) 8

Problem 7.
$f(x) = \frac{1}{x+2}$, $D_f : x \neq -2$.
$x = -2$ is NOT a removable point of discontinuity.

Problem 8.
The function is continuous on $(-\infty, \infty)$.

Problem 9.
$f(x) = \begin{cases} -x^2 - 1 & , x > 0 \\ 5x - 1 & , x < 0 \end{cases}$. $x = 0$ is a removable point of discontinuity, since we can define $f(x)$ to be -1 at $x = 0$, making our function continuous on $(-\infty, \infty)$

Problem 15.

(a) Yes

(b) Yes, $f'(0) = 0$

Section 8.1 Local Linearity and the Derivative

Problem 1.

(a) $f(x) = \sqrt{x}$, $x = 25$.

Tangent line at $x = 25$ is $y - 5 = \frac{1}{2\sqrt{25}}(x - 25)$ or $y = 5 + 0.1(x - 25)$. So the linearization is $\sqrt{23} = f(23) \approx 5 + 0.1(23 - 25) \approx 4.8$. Using calculator we check that $\sqrt{23} \approx 4.795$

(b) $\sqrt{24} \approx f(24) = 0.1(24 - 25) + 5 = 4.9$; This is an overestimate, and using calculator we check that $|4.9 - \sqrt{24}| \approx |4.9 - 4.89898| = 0.00102$.

(c) $\sqrt{24.9} \approx f(24.9) = 0.1(24.9 - 25) + 5 = 4.99$; This is an overestimate, and using calculator we check that $|4.99 - \sqrt{24.9}| \approx |4.99 - 4.98999| = 0.00001$.

(d) $\sqrt{25.1} = f(25.1) \approx 5 + 0.1(25.1 - 25) \approx 5.01$. Using calculator we check that $\sqrt{25.1} \approx 5.01$

(e) $\sqrt{26} \approx f(26) = 0.1(26 - 25) + 5 = 5.1$; This is an overestimate, and using calculator we check that $|5.1 - \sqrt{26}| \approx |5.1 - 5.09902| = 0.00098$. $|5.1 - \sqrt{26}| \approx |5.1 - 5.09902| = 0.00098$.

(f) $\sqrt{27} = f(27) \approx 5 + 0.1(27 - 25) \approx 5.2$. Using calculator we check that $\sqrt{27} \approx 5.196$

Problem 5.

$f(x) = \sqrt[3]{x}$, $x = 27$.

The slope of the tangent to $f(x)$ is $(\frac{1}{3}x^{-\frac{2}{3}})|_{x=27} = \frac{1}{27}$. So, the tangent is $y - 3 = \frac{1}{27}(x - 27)$ or $y = 3 + \frac{1}{27}(x - 27)$.
Hence $\sqrt[3]{30} \approx 3 + \frac{1}{27}(30 - 27) \approx 3.111$
Using calculator we check that $\sqrt[3]{30} \approx 3.107$