## Section 8.3 Derivatives of Sums, Products, Quotients, and Power Functions

**Problem 1.**

$f'(x) = 6x + 3 - 3x^{-2} - 6x^{-3}$

**Problem 2.**

$f(x) = \frac{1}{5}(x - 2x^2) \Rightarrow f'(x) = \frac{1}{5}(1 - 4x)$

**Problem 3.**

Product Rule. $f'(x) = \pi[(6x + 7)(x - 2) + (3x^2 + 7x + 1)] = \pi[6x^2 - 12x + 7x - 14 + 3x^2 + 7x + 1] = \pi[9x^2 + 2x - 13]$

**Problem 4.**

Quotient Rule. $f'(x) = \frac{0\cdot(x^2+4)-1(2x)}{(x^2+4)^2} = \frac{-2x}{(x^2+4)^2}$

**Problem 5.**

Quotient Rule. $f'(x) = \frac{(x+2)(1)-(x)(1)}{(x+2)^2} = \frac{2}{(x+2)^2}$

**Problem 6.**

$f(x) = \frac{x+2}{x} = 1 + 2x^{-1}, \quad f'(x) = -2x^{-2} = \frac{-2}{x^2}$

**Problem 7.**

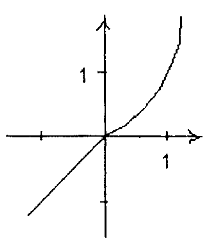
$f(x) = \left(\frac{5}{2}x^2 + 7x^5 - 5x\right)x = \frac{5}{2}x^3 + 7x^6 - 5x^2 \Rightarrow f'(x) = \frac{15}{2}x^2 + 42x^5 - 10x.$

**Problem 8.**

$f(x) = ax^{-1} + bcx - bdx^2, \quad f'(x) = -ax^{-2} + bc - 2bdx = -\frac{a}{x^2} + b(c - 2dx)$

**Problem 14.**

(a)




(b) $f'(x) = \begin{cases} 2x & ,x > 0 \\ 1 & ,x < 0 \\ undefined & ,x = 0 \end{cases}$

**Problem 17.**

$$f(x) = \begin{cases} x^3 & ,x \le 1 \\ kx & ,x > 1 \end{cases}$$

(a) To make $f$ continuous at $x = 1$ we have to choose $k$ such that $x^3 = kx$ when $x = 1$. So $k = 1$.

(b) $f$ is not differentiable at $x = 1$.

**Problem 20.**

$$\frac{d}{dx}\left(\frac{x+1}{x^3+3x+1}\right) = \frac{x^3+3x+1-(x+1)(3x^2+3)}{(x^3+3x+1)^2} = \frac{x^3+3x+1-3x^3-3x-3x^2-3}{(x^3+3x+1)^2} = \frac{-2x^3-3x^2-2}{(x^3+3x+1)^2}$$

**Problem 21.**

$$\frac{d}{dx}\left(\frac{\pi}{\pi x+\pi}\right) = \frac{d}{dx}\left(\frac{1}{x+1}\right) = \frac{(x+1)(0)-(1)(1)}{(x+1)^2} = -\frac{1}{(x+1)^2}.$$

**Problem 22.**

$$\frac{d}{dx}\left(\frac{2x^2+x+1}{\sqrt{2x}}\right) = \frac{(\sqrt{2x})(4x+1)-\left(\frac{1}{\sqrt{2x}}\right)(2x^2+x+1)}{2x} = \frac{(2x)(4x+1)-(1)(2x^2+x+1)}{2\sqrt{2}x^{3/2}} =$$
$$\frac{8x^2+2x-2x^2-x-1}{2\sqrt{2}x^{3/2}} = \frac{\sqrt{2}(6x^2+x-1)}{4x^{3/2}}.$$

**Problem 23.**

$$\frac{d}{dx}\left(\frac{x^2+5x}{2x^{10}}\right) = \frac{d}{dx}\left(\frac{1}{2}x^{-8}+\frac{5}{2}x^{-9}\right) = -4x^{-9}-\frac{45}{2}x^{-10} = -\frac{4}{x^9}-\frac{45}{2x^{10}}$$

SECTION 9.1 EXPONENTIAL GROWTH

**Problem 2.**

(a) Let $G(s)$ = the number of grains as a function of the $s^{th}$ square. Thus $G(s) = 2^{s-1}$, and hence $G(64) = 2^{63} \approx 9.22 \times 10^{18}$ grains of rice were allocated to the 64th square.

(b) Let $W(s)$ = weight of rice at square $s$ measured in grams. Thus $W(s) = 0.02G(s) = 0.02(2^{s-1})$, and hence $W(64) = 0.02(2^{63}) \approx 0.02(9.22 \times 10^{18}) \approx 1.844 \times 10^{17}$ grams, which is about $(1.844 \times 10^{17})/(907.18 \times 1000) \approx 2.03 \times 10^{11}$ tons of rice. This is about 500 times the world production of rice in 1980.

(c)

| squares | | total grains |
|---|---|---|
| 1 | | $1 = 2^1 - 1$ |
| 2 | $1+2=$ | $3 = 2^2 - 1$ |
| 3 | $1+2+4=$ | $7 = 2^3 - 1$ |
| 4 | $1+2+4+8=$ | $15 = 2^4 - 1$ |
| . | | . |
| . | | . |
| 64 | | $2^{64} - 1 \approx 1.84 \times 10^{19}$ grains |

So the total mass is about $3.69 \times 10^{17}$ grams.