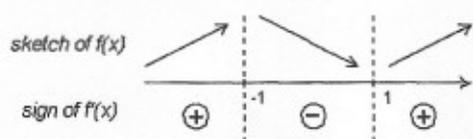


**Problem 1.**

(a)  $f'(x) = 3x^2 - 3$ .  $f'(x) = 0 \Leftrightarrow 3x^2 - 3 = 0$ , so  $x^2 - 1 = 0$  and therefore  $x = 1$  or  $x = -1$  are critical points.

(b)



This tells us that at  $x = -1$   $f$  has a local maximum and at  $x = 1$   $f$  has a local minimum.

(c) There are no absolute minimum or maximum values.

**Problem 2.**

(a) From problem 1, we have that  $x = -1$  and  $x = 1$  are critical points. The endpoints of the domain  $[-5, 5]$ ,  $x = -5$  and  $x = 5$ , are also critical points.

(b) From problem 1,  $x = -1$  is a local maximum point and  $x = 1$  is a local minimum point. The absolute minimum occurs at  $x = -5$ , and the absolute maximum occurs at  $x = 5$ .

(c) The absolute minimum value is  $f(-5) = -108$ , and the absolute maximum value is  $f(5) = 112$ .

**Problem 3.**

(a) From problem 1, we have that  $x = 1$  is a critical point. The endpoints of the domain,  $x = 0$  and  $x = 3$ , are also critical points.

(b) The point  $x = 0$  is neither a local maximum or minimum point nor an absolute minimum or maximum point. From problem 1,  $x = 1$  is a local minimum point and the absolute minimum point. The point  $x = 3$  is the absolute maximum point.

(c) The absolute minimum value is  $f(1) = 0$ , and the absolute maximum value is  $f(3) = 20$ .

**Problem 4.**

(a) From problem 1, we have that  $x = 1$  is a critical point.

(b) As in problem 3,  $x = 1$  is a local minimum point and the absolute minimum point.

(c) The absolute minimum value is  $f(1) = 0$ . There is no absolute maximum value.

**Problem 7.**

(a)  $f'(x) = 5x^4 - 20 = 5(x^4 - 4) \Rightarrow 0 = f'(x) = 5(x^2 - 2)(x^2 + 2) \Rightarrow x = -\sqrt{2}$  and  $x = \sqrt{2}$  are the critical points.

(b)  $f'(x)$  is positive on  $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$  and is negative on  $(-\sqrt{2}, \sqrt{2})$ . The first derivative test implies that  $x = -\sqrt{2}$  is a local maximum point and  $x = \sqrt{2}$  is a local minimum point. As  $f(x)$  increases without bound as  $x$  increases without bound,  $x = -\sqrt{2}$  is not an absolute maximum point. As  $f(x)$  decreases without bound as  $x$  decreases without bound,  $x = \sqrt{2}$  is not an absolute minimum point.

(c) There are no absolute minimum or maximum values.

**Problem 8.**

(a) From problem 7,  $x = -\sqrt{2}$  is a critical point. The endpoints of the domain  $[-2, 0]$ ,  $x = -2$  and  $x = 0$ , are also critical points.

(b) The point  $x = -2$  is neither a local maximum or minimum point nor an absolute minimum or maximum point. The point  $x = -\sqrt{2}$  is a local and absolute maximum point. The point  $x = 0$  is the absolute minimum point.

(c) The absolute minimum value is  $f(0) = 5$ , and the absolute maximum value is  $f(-\sqrt{2}) = 16\sqrt{2} + 5$ .

**Problem 9.**

- (a) From problem 7,  $x = \sqrt{2}$  is a critical point. The endpoints of the domain,  $x = 0$  and  $x = 2$ , are also critical points.
- (b) The point  $x = 0$  is the absolute maximum point. The point  $x = \sqrt{2}$  is a local and absolute minimum point. The point  $x = 2$  is neither a local maximum or minimum point nor an absolute minimum or maximum point.
- (c) The absolute minimum value is  $f(\sqrt{2}) = 5 - 16\sqrt{2}$ , and the absolute maximum value is  $f(0) = 5$ .

**Problem 18.**

- (a)  $f'(x) = 2xe^{-x} + x^2(-e^{-x}) = -x(x-2)e^{-x}$ ;  $0 = f'(x) \Rightarrow x = 2$  and  $x = 0$  are the critical points.
- (b)  $f'(x)$  is positive on  $(0, 2)$  and is negative on  $(-\infty, 0) \cup (2, \infty)$ . The first derivative test implies that  $x = 0$  is a local minimum point and  $x = 2$  is a local maximum point.
- (c) No. The function values of  $f$  increase without bound as  $x$  decreases without bound.
- (d) Yes, the absolute minimum value of  $f(0) = 0$  is achieved at  $x = 0$ .

10.2

**Problem 1.**

- (a)  $f'(x) = 3x^2 - 6 = 0$ ,  $3(x^2 - 2) = 0$ , so  $x^2 = 0 \Rightarrow x = \sqrt{2}$  and  $x = -\sqrt{2}$  are critical points.
- (b) Second derivative  $f''(x) = 6x$ . Since  $f''(-\sqrt{2}) = -6\sqrt{2} < 0$ ,  $f$  has a local maximum at  $x = -\sqrt{2}$ . Since  $f''(\sqrt{2}) = 6\sqrt{2} > 0$ ,  $f$  has a local minimum at  $x = \sqrt{2}$ .