

11.1

Problem 3.Answers will vary; one possibility is: $f(x) = -(x-1)^3$.**Problem 4.**Answers will vary; two possibilities are: $f(x) = \frac{1}{3}x^3 - x^2$ and $f(x) = (x+1)(x-2)^2$.**Problem 5.**Answers will vary; one possibility is: $f(x) = (x-1)^3 = x^3 - 3x^2 + 3x - 1$.**Problem 6.**Answers will vary; one possibility is: $f(x) = x^3 + x$.**Problem 7.**Answers will vary; one possibility is: $f(x) = -\frac{2}{27}(x-3)^3$. As f is always decreasing, f must have exactly one zero. Hence, f is of the form $f(x) = k(x-3)^3$, where $k < 0$ is a constant. Using $f(0) = 2$, we have that $2 = k(0-3)^3$, from which we obtain $k = -\frac{2}{27}$.**Problem 12.**

- (a) $f(x) = x^3 - 3x + 1 \Rightarrow f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x+1)(x-1)$. $0 = f'(x) \Rightarrow x = -1$ and $x = 1$ are the critical points. Now $f''(x) = 6x$ gives $f''(-1) = -6 < 0$ and $f''(1) = 6 > 0$. Therefore, $x = -1$ is a local maximum point and $x = 1$ is a local minimum point. Moreover neither of these critical points are absolute extreme points because $\lim_{x \rightarrow \infty} f(x) = \infty$ and $\lim_{x \rightarrow -\infty} f(x) = -\infty$.
- (b) $f(x) = x^3 + 3x + 1 \Rightarrow f'(x) = 3x^2 + 3 = 3(x^2 + 1)$. $0 = f'(x) \Rightarrow x^2 = -1$. Hence there are no real values of x for which $0 = f'(x)$ and consequently no critical points.

Problem 13.

- (a) $f(x) = -x^3 - 3x^2 + 9x + 5 \Rightarrow f'(x) = -3x^2 - 6x + 9 = -3(x+3)(x-1)$. Now $0 = f'(x) \Rightarrow x = -3$ or $x = 1$. Now $f''(x) = -6x - 6$, and hence $f''(-3) = 12 > 0$ and $f''(1) = -12 < 0$. Thus there is a local minimum at $x = -3$ and a local maximum at $x = 1$. As $\lim_{x \rightarrow \infty} f(x) = -\infty$ and $\lim_{x \rightarrow -\infty} f(x) = \infty$, neither of the local extrema are absolute extrema.
- (b) $f(x) = x^3 + 3x^2 + 9x + 8 \Rightarrow f'(x) = 3x^2 + 6x + 9 = 3(x^2 + 2x + 3)$. Now $0 = f'(x) \Rightarrow x = \frac{-2 \pm \sqrt{2^2 - 4(1)(3)}}{2}$. Hence there are no real values of x for which $0 = f'(x)$ and consequently no critical points.

11.2

Problem 2.Answers will vary; one possibility: $P(x) = (x-1)(x+3)$.**Problem 3.**Answers will vary; one possibility: $P(x) = x(x+1)(x-5)$.**Problem 4.**Answers will vary; one possibility: $P(x) = x^4 + 5$.**Problem 5.**Answers will vary; one possibility: $P(x) = (x - \sqrt{2})^4$.**Problem 6.**Answers will vary; one possibility: $P(x) = x(x-9)^2(x-3)(x+e)$.**Problem 7.**Answers will vary; one possibility: $P(x) = -\frac{(x-\pi-1)^3}{(\pi+1)^3}$.**Problem 8.**Answers will vary; one possibility: $P(x) = (x+3)^4 + 2$.**Problem 9.** $P(x) = k(x-2)^2(x+3)^2$, for some $k \neq 0$. Now $-2 = P(0) = k(4)(9) = 36k \Rightarrow k = -\frac{1}{18}$. This answer is unique.

Problem 15.

(a) $f(x) = 2x^3 + 2x^2 - 12x = 2x(x + 3)(x - 2)$. The zeros are $x = -3, 0$, and 2 .

(b) $g(x) = 2x^3 + 2x^2 + 12x = 2x(x^2 + x + 6)$. Now if $x^2 + x + 6 = 0$, then $x = \frac{-1 \pm \sqrt{1-4(6)}}{2} = \frac{-1 \pm \sqrt{-23}}{2}$, which are not real solutions. The only zero is $x = 0$.

Problem 17.

(a) $P(x) = x^3 - x^2 - 4x + 4 = (x^2 - 4)(x - 1) = (x + 2)(x - 2)(x - 1)$. The zeros are $x = -2, 1$, and 2 .

(b) $Q(x) = x^3 - x^2 + 4x - 4 = (x - 1)(x^2 + 4)$. Now $x^2 + 4 = 0$ has no real solutions; hence, the zero is $x = 1$.

Problem 22.

(a) Definitely false. A polynomial that is symmetric about the origin has odd degree.

(b) Definitely true. A polynomial that is symmetric about the origin has odd degree.

(c) Definitely false. A polynomial that is symmetric about the origin has odd degree, and odd-degree polynomials have at least one zero.

(d) Definitely true. A polynomial that is symmetric about the origin has odd degree, and odd-degree polynomials have at least one zero.

(e) Definitely true. Because $P(x)$ is symmetric about the origin, $(0, 0)$ cannot be a turning point, and if $(a, P(a))$ is a turning point, $(-a, -P(a))$ is also a turning point. Hence the number of turning points is a (nonnegative) even number.

Problem 23.

(a) Definitely false. An even-degree polynomial can be symmetric about the y -axis but not the origin.

(b) Possibly true. True if $P(x) = x^4$; false if $f(x) = x^4 + x^3$.

(c) Definitely true. $\lim_{x \rightarrow -\infty} P(x) = \lim_{x \rightarrow \infty} P(x) = L$, where $L = \pm\infty$. As $P(0) = 0$, $P(x)$ must have at least one turning point.

(d) Definitely false. $\lim_{x \rightarrow -\infty} P(x) = \lim_{x \rightarrow \infty} P(x) = L$, where $L = \pm\infty$, which implies that $P(x)$ must have an odd number of turning points.

(e) Definitely true. $\lim_{x \rightarrow -\infty} P(x) = \lim_{x \rightarrow \infty} P(x) = L$, where $L = \pm\infty$, which implies that $P(x)$ must have an odd number of turning points.

11.3

Problem 2.

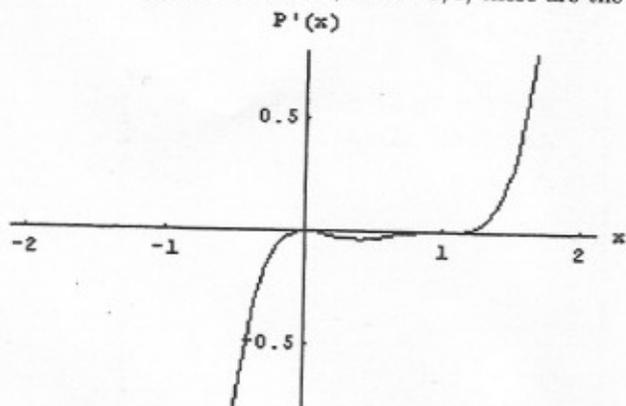
(a) Since $P'(x)$ is degree 5, P is degree 6.

(b) The critical points of P occur whenever $P'(x) = 0$, that is at $x = 0, -2$.

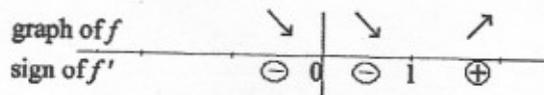
(c) Because $\lim_{x \rightarrow -\infty} P'(x) = -\infty$, $\lim_{x \rightarrow \infty} P'(x) = \infty$, and $P'(x)$ is continuous everywhere, $P(x)$ has an absolute minimum. We know it must be obtained at one of the critical points, so we calculate $P''(x) = 2x(x + 2)^3 + 3x^2(x + 2)^2$; unfortunately, $P''(0) = 0$ and $P''(-2) = 0$, so the second derivative test lends no information. (How could you have predicted that?) Instead we look at the sign of $P'(x)$; across $x = 0$ the sign remains positive, but across $x = -2$ the sign changes from negative to positive. Thus, there is an absolute minimum at $x = -2$. However, we cannot find the value of the function here since we don't have an expression for the function; we can determine only the value of the function relative to some other value of the function.

Problem 4.

- (a) Note that $P'(x)$ has two zeros, at $x = 0, 1$; these are the critical points of $P(x)$.

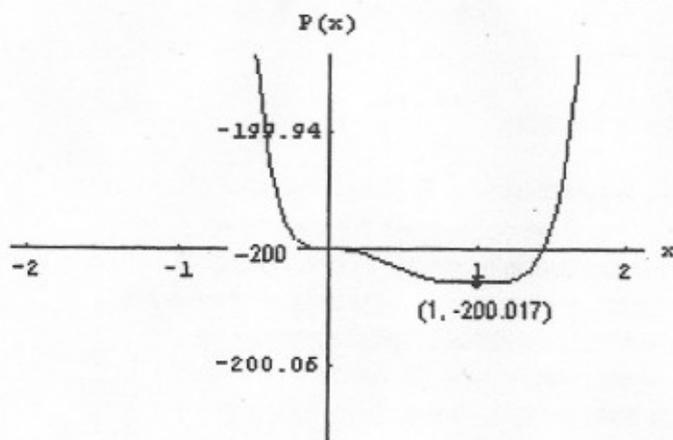


- (b)



Note that $P' > 0$ on $(1, \infty)$ and $P' < 0$ on $(-\infty, 0) \cup (0, 1)$.

- (c) Because $P(x) < 0$ at $x = 0$, $P(x) < 0$ at $x = 1$ since P is decreasing on the interval. Since $P'(x)$ is positive and increasing for $x > 1$, $P(x)$ must be increasing at an increasing rate for $x > 1$; because $P(x) < 0$ at $x = 1$, $P(x)$ must cross the x -axis.



Problem 5.

- (a) False; (b) True; (c) Possible; (d) True; (e) False; (f) False; (g) True; (h) False.

Problem 9.

- (a) The graph has the general shape of the graph of a fourth-degree polynomial function. There is a even order zero at $x = a$ and odd order zeros at $x = b$ and $x = c$. Hence the function has the form $P(x) = k(x - a)^2(x - b)(x - c)$, for some constant k . As the y -intercept is d , we have $d = P(0) = k(0 - a)^2(0 - b)(0 - c) = ka^2bc \Rightarrow k = \frac{d}{a^2bc}$. Therefore, $P(x) = \frac{d}{a^2bc}(x - a)^2(x - b)(x - c)$
- (b) The graph has the general shape of the graph of a sixth-degree polynomial function. There are a even order zeros at $x = a$ and $x = c$ and odd order zeros at $x = b$ and $x = 0$. Hence the function has the form $P(x) = kx(x - a)^2(x - b)(x - c)^2$, for some constant k . As the graph contains the point $(-1, 3)$, we have $3 = P(-1) = k(-1)(-1 - a)^2(-1 - b)(-1 - c)^2 \Rightarrow k = \frac{3}{(a+1)^2(b+1)(c+1)^2}$. Therefore, $P(x) = \frac{3}{(a+1)^2(b+1)(c+1)^2}x(x - a)^2(x - b)(x - c)^2$.

Problem 1

- (a) The graph has a single simple zero at $x = 0$ and a vertical asymptote at $x = -1$. As the sign of y changes across the vertical asymptote, there is an odd power of $(x + 1)$ in the denominator of the function. As there is a horizontal asymptote at $y = 2$, the degrees of the numerator and denominator of the function are equal, and the lead coefficient of the numerator is 2. Hence $y = \frac{2x}{x+1} = 2 - \frac{2}{x+1}$.
- (b) The graph has simple zeros at $x = -2$ and $x = 0$ and a vertical asymptote at $x = -1$. As the sign of y does not change across the vertical asymptote, there is an even power of $(x + 1)$ in the denominator of the function. As there is a horizontal asymptote at $y = 2$, the degrees of the numerator and denominator of the function are equal, and the lead the coefficient of the numerator is 2. Hence $y = \frac{2x(x+2)}{(x+1)^2} = -\frac{2}{(x+1)^2} + 2$.

Teacher Will Do	Student Will Do	Activity
<ul style="list-style-type: none"> During a free period, such as break time or the end of the day, the teacher will place several examples of machines around the classroom. Examples could be pickles, egg beater, hot clipper, etc. 	<ul style="list-style-type: none"> Students will be asked to spend some time observing each of the machines and thinking about the parts that make the machine work. This is distinct from the parts of the machine. For example a seat is a part of a bicycle, but it has little effect on the action of the bike. 	Thinking about Machines
	<ul style="list-style-type: none"> Students will be asked to read section 4.4 in Motion, Forces, and Energy pages 95-96. This section gives information on the incline plane, wedge, and screw. 	Homework

Teacher Will Do	Student Will Do	Activity
<ul style="list-style-type: none"> The teacher will provide students with the attached chart. The teacher will circulate among the students during the reading activity to identify any children who are struggling. When students are finished the teacher will go over class with the students to identify any misconceptions or problems. 	<ul style="list-style-type: none"> The students will work in small groups of 3 or 4 to fill out the chart. They will be instructed to try and fit out the chart from memory first, and then to go back to the reading to find anything they may have forgotten. 	Reading Activity
<ul style="list-style-type: none"> The teacher will bring the examples of machines from yesterday back out. After examples have been provided for each of the types of simple 	<ul style="list-style-type: none"> Students will have 15 minutes to identify as many of each kind of simple machine as possible in the classroom. Each simple machine will be listed on the board and students will be asked 	Simple Machines Scavenger Hunt