**Problem 1.** |11.4|

(c) The graph has no zeros and always lies above the x-axis. There is a vertical asymptote at $x = -1$, which, in this case, implies that there is an even power of $(x+1)$ in the denominator of the function. As there is a horizontal asymptote at $y = 0$, the degree of the numerator of the function is less than the degree of the denominator of the function. Hence $y = \frac{2}{(x+1)^2}$.

(d) The graph has no zeros and vertical asymptotes at $x = -1$ and $x = 2$. The sign of y changes across both of these vertical asymptotes, which implies that there are odd powers of $(x+1)$ and $(x-2)$ in the denominator of the function. As there is a horizontal asymptote at $y = 0$, the degree of the numerator of the function is less than the degree of the denominator of the function. Hence the equation has the form $y = \frac{k}{(x+1)(x-2)}$, where k is a nonzero constant. As $y < 0$ for $|x| > 2$, $k < 0$. For simplicity, we choose $k = -1$. Therefore, $y = -\frac{1}{(x+1)(x-2)}$.

(e) The graph has no zeros and vertical asymptotes at $x = -1$ and $x = 2$. The sign of y changes across $x = -1$ but does not change across $x = 2$. Thus, there is an odd power $(x+1)$ and an even power of $(x-2)$ in the denominator of the function. As there is a horizontal asymptote at $y = 0$, the degree of the numerator of the function is less than the degree of the denominator of the function. Hence the equation has the form $y = \frac{k}{(x+1)(x-2)^2}$, where k is a nonzero constant. As $y < 0$ for $x > 2$, $k < 0$. For simplicity, we choose $k = -1$. Therefore, $y = -\frac{1}{(x+1)(x-2)^2}$

(f) The graph has no zeros, lies above the x-axis and has vertical asymptotes at $x = -3$ and $x = 1$. Thus, there are even powers of $(x+3)$ and $(x-1)$ in the denominator of the function. As there is

a horizontal asymptote at $y = 1$, the equation has the form $y = \frac{1}{(x+3)^2(x-1)^2} + 1 = \frac{1+(x+3)^2(x-1)^2}{(x+3)^2(x-1)^2} = \frac{x^4+4x^3-2x^2-12x+10}{x^4+4x^3-2x^2-12x+9}$.

(g) The graph has no zeros, lies above the x-axis and has vertical asymptotes at $x = -3$ and $x = 1$. Thus, there are even powers of $(x+3)$ and $(x-1)$ in the denominator of the function. As there is a horizontal asymptote at $y = 0$, the degree of the numerator of the function is less than the degree of the denominator of the function. Hence the equation has the form $y = \frac{k}{(x+3)^2(x-1)^2}$. As $y > 0$ for $x > 1$, $k > 0$, and, for simplicity, we choose $k = 1$. Therefore, the equation is $y = \frac{1}{(x+3)^2(x-1)^2}$.

(h) The graph has no zeros and a vertical asymptote at $x = 0$. As the y changes sign across $x = 0$, there is an odd power x in the denominator of the function. As there are no vertical asymptotes, the degree of the numerator of the function is greater than the degree of the denominator. Hence the equation $y = \frac{x^2+1}{x}$ suffices.
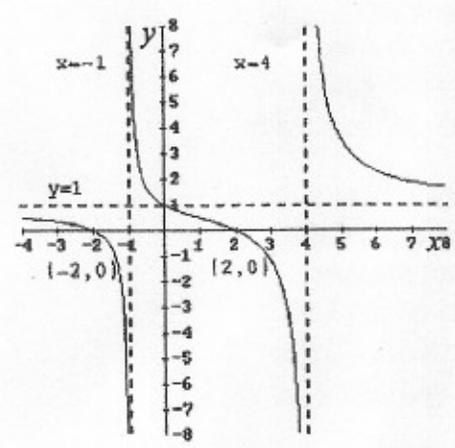
Problem 4.

(a)
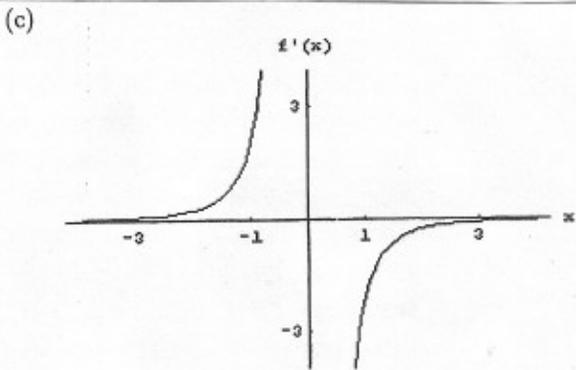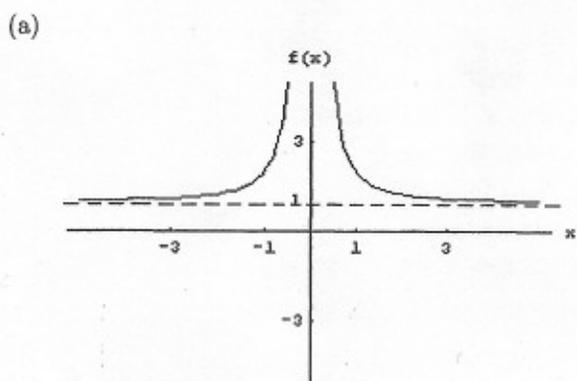
(b) (i) $\lim_{x \to \infty} f(x) = 1$;

(ii) $\lim_{x \to 0} f(x) = \infty$;

(iii) $\lim_{x \to \infty} f'(x) = 0$;

(iv) $\lim_{x \to 0^+} f'(x) = -\infty$;

(v) $\lim_{x \to 0^-} f'(x) = \infty$.

(c)

(d) $f(x) = 1 + \frac{1}{x^2} \Rightarrow f'(x) = -\frac{2}{x^3} \Rightarrow f''(x) = \frac{6}{x^4}$.

(e) Yes, the answers are consistent. This is a good place for the students to compare the graphical data with the analytic calculations.
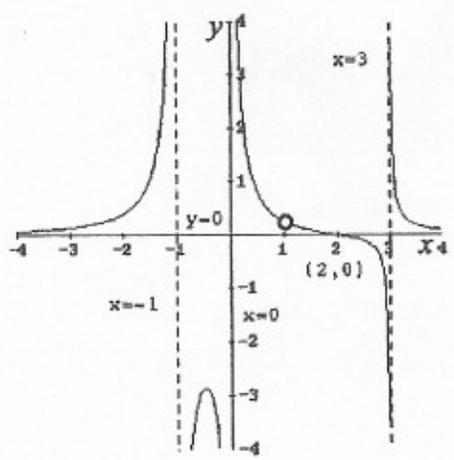
Problem 7.

(a) Note that $f(x) = \frac{x^2-4}{x^2-3x-4} = \frac{(x+2)(x-2)}{(x+1)(x-4)}$. The x-intercepts are $x = \pm 2$; the y-intercept is $y = 1$; the vertical asymptotes are $x = -1, 4$; the horizontal asymptote is $y = 1$.

(b) The x-intercept is the origin; the y-intercept is the origin; the vertical asymptote is $x = 1$; the horizontal asymptote is $y = 3$.
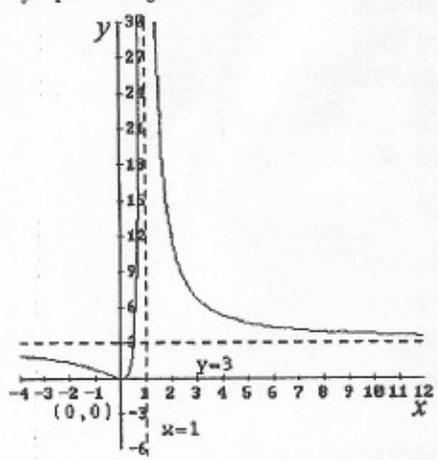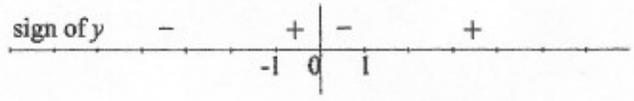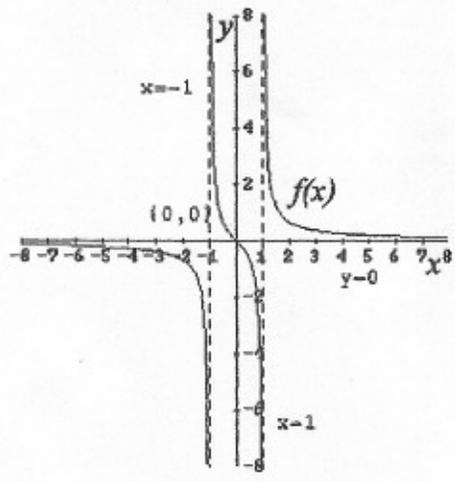

(c) The x-intercept is $x = 2$; there are no y-intercepts; the vertical asymptotes are $x = -1, 0, 3$; the horizontal asymptote is the x-axis; there is a hole at $\left(1, \frac{1}{3}\right)$.
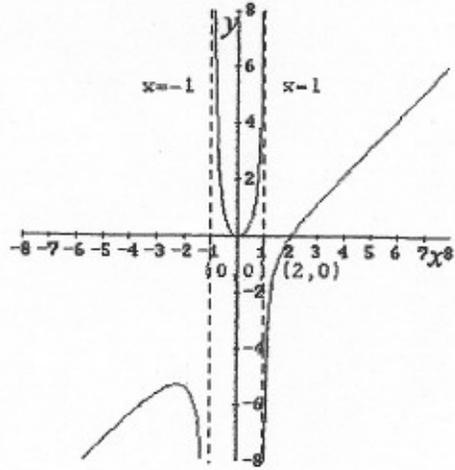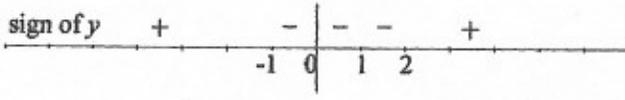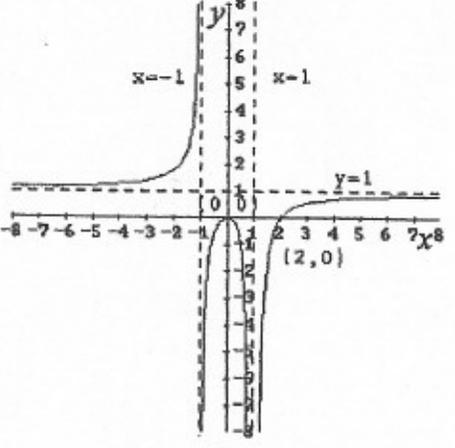
Problem 12.

(i) The x-intercept is the origin; the y-intercept is the origin; the vertical asymptotes are $x = \pm 1$; the horizontal asymptote is the x-axis.
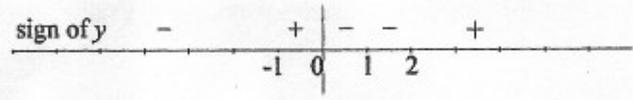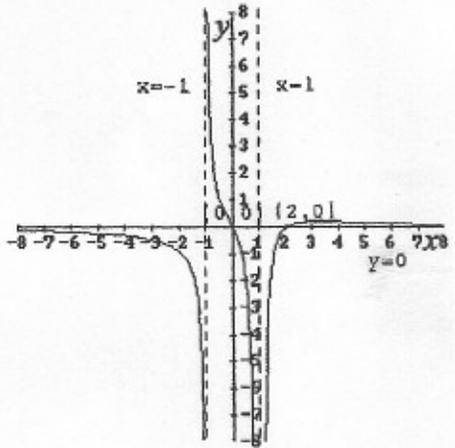

(ii),(iii) The x-intercepts are $x = 0, 2$; the y-intercept is the origin; the vertical asymptotes are $x = \pm 1$; there are no horizontal asymptotes.
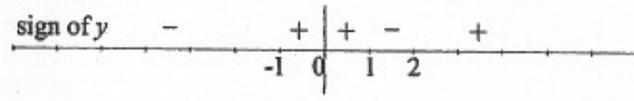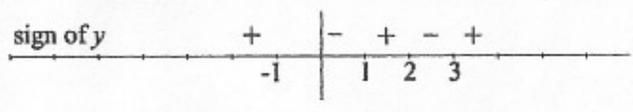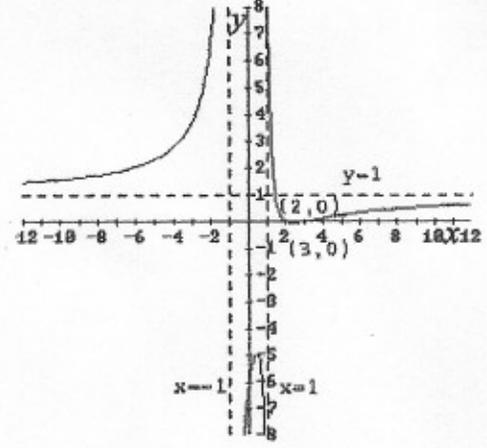

(iv) The x-intercepts are $x = 0, 2$; the y-intercept is the origin; the vertical asymptotes are $x = \pm 1$; the horizontal asymptote is $y = 1$.
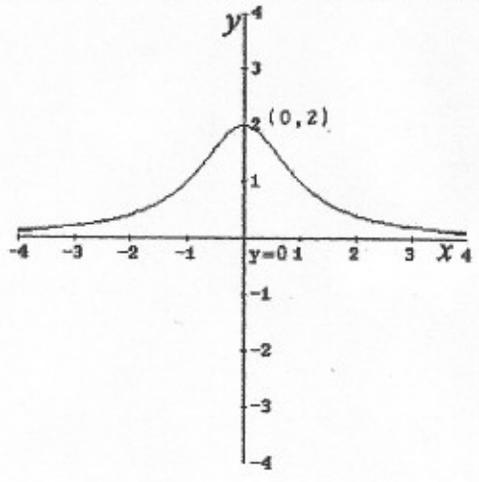
(v) The x-intercepts are $x = 0, 2$; the y-intercept is the origin; the vertical asymptotes are $x = \pm 1$; the horizontal asymptote is the x-axis.


(vi) The x-intercepts are $x = 2, 3$; the y-intercept is $y = -6$; the vertical asymptotes are $x = \pm 1$; the horizontal asymptote is $y = 1$.

(vii) There are no x-intercepts; the y-intercept is $y = 2$; there are no vertical asymptotes; the horizontal asymptote is the x-axis. Note that $y < 0$ for all values of x.

(viii) The x-intercept is the origin; the y-intercept is the origin; there are no vertical asymptotes; the horizontal asymptote is $y = -1$. Note that $y < 0$ for all values of x.
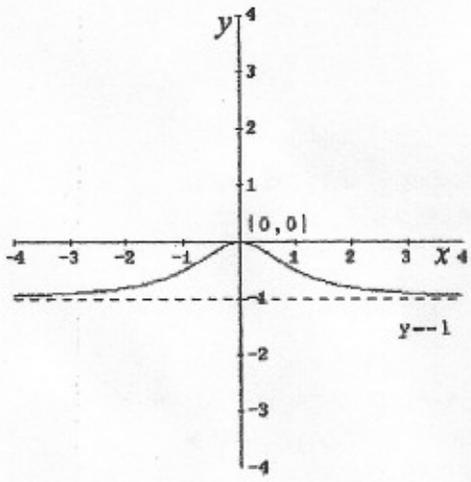
$\boxed{12.1}$

Problem 1.

(a) 1-to-1 and invertible (as long as social security numbers are not reassigned after someone dies)

(b) Not 1-to-1 and hence not invertible; five people are put into each group.

(c) Not 1-to-1 and not invertible; many sites have the same altitude.