

Problem 1.

- (a) $3^{\log_3 2} = 2$.
- (b) $\log x + \log x^2 - 3 \log x = \log x + 2 \log x - 3 \log x = 0$.
- (c) $2 \log(x+3) - 3 \log(x+3) + \log(10^{\sqrt{7}}) = -\log(x+3) + \sqrt{7}$.
- (d) $10^{\log x^2} = x^2$.
- (e) $10^{3 \log x} = 10^{\log x^3} = x^3$.
- (f) $10^{-\log x} = 10^{\log x^{-1}} = x^{-1} = \frac{1}{x}$.
- (g) $10^{-0.5 \log x} = 10^{\log x^{-0.5}} = x^{-0.5} = \frac{1}{\sqrt{x}}$.
- (h) $3^{-\log_3(x+y)} = 3^{\log_3(x+y)^{-1}} = (x+y)^{-1} = \frac{1}{x+y}$.
- (i) $2^{(\log_2 10 - \log_2 5)} = 2^{(\log_2 \frac{10}{5})} = 2^{\log_2 2} = 2$.
- (j) $10^{\frac{\log x}{2}} = (10^{\log x})^{\frac{1}{2}} = x^{\frac{1}{2}} = \sqrt{x}$.

Problem 9.

- (a) $10^{\frac{\log 8+1}{2}} = (10^{\log 8+1})^{\frac{1}{2}} = \sqrt{10^{\log 8} 10^1} = \sqrt{8(10)} = \sqrt{80} = 4\sqrt{5}$.
- (b) $e^{-\frac{\ln 8}{3}+2} = e^{-\frac{\ln 8}{3}} e^2 = e^{-\frac{1}{3} \ln 8} e^2 = (e^{\ln 8})^{-\frac{1}{3}} e^2 = 8^{-\frac{1}{3}} e^2 = \frac{e^2}{\sqrt[3]{8}} = \frac{e^2}{2}$.

Problem 11.

$$\frac{\log 12}{2} = \frac{1}{2} \log(2^2 3) = \frac{1}{2} (\log 2^2 + \log 3) = \frac{1}{2} (2 \log 2 + \log 3) = \frac{1}{2} (2a + b) = a + \frac{b}{2}$$

Problem 13.

$$\log(9\sqrt{2}) = \log 9 + \log \sqrt{2} = \log 3^2 + \log 2^{\frac{1}{2}} = 2 \log 3 + \frac{1}{2} \log 2 = 2b + \frac{a}{2}$$

Problem 15.

$$\ln \sqrt{x} - \frac{\ln x^3}{2} - 3 \ln x = \ln \sqrt{x} - \ln(x^3)^{\frac{1}{2}} - \ln x^3 = \frac{1}{2} \ln x - \frac{3}{2} \ln x - 3 \ln x = -4 \ln x = \ln \left(\frac{1}{x^4}\right)$$

Problem 16.

$$a \ln(x+3) - b \ln\left(\frac{1}{x}\right) - c \ln(x+1) = \ln(x+3)^a + \ln\left(\frac{1}{x}\right)^{-b} + \ln(x+1)^{-c} = \ln(x+3)^a \left(\frac{1}{x}\right)^{-b} (x+1)^{-c} = \ln \frac{(x+3)^a x^b}{(x+1)^c}$$

Problem 2.

- (a) $\log(10^{2x}) = \log 93 \Rightarrow 2x = \log 93 \Rightarrow x = \frac{\log 93}{2} (\approx 0.984)$.
- (b) $\log 10^{3x+2} = \log 1,000,000 \Rightarrow 3x+2 = 6 \Rightarrow x = \frac{4}{3}$.
- (c) $x+1 = \log_2 7 \Rightarrow x = \log_2(7) - 1 (\approx 1.807)$.
- (d) $3^{x+x^2} = 3^1 \Rightarrow \log_3(3^{x+x^2}) = \log_3(3^1) \Rightarrow x+x^2 = 1 \Rightarrow x = \frac{-1 \pm \sqrt{5}}{2} (\approx 0.618, -1.618)$.
- (e) $\ln(5B^x) = \ln((2C)^{x+1}) \Rightarrow x \ln(5B) = (x+1) \ln(2C) \Rightarrow x(\ln(5B) - \ln(2C)) = \ln(2C) \Rightarrow x = \frac{\ln(2C)}{\ln(5B) - \ln(2C)}$.
- (f) $\ln x = 3 \Rightarrow e^{\ln x} = e^3 \Rightarrow x = e^3 (\approx 20.086)$.
- (g) $10^{\log_{10} x} = 10^{17} \Rightarrow x = 10^{17}$.
- (h) $e^{\ln(5x-40)} = e^3 \Rightarrow 5x-40 = e^3 \Rightarrow x = \frac{e^3+40}{5} (\approx 12.017)$.
- (i) $10^{\log_{10}(2x^2+4)} = 10^2 \Rightarrow 2x^2+4 = 100 \Rightarrow x = \pm 4\sqrt{3} (\approx \pm 6.928)$.
- (j) $2^{x/7} = \frac{16}{3} \Rightarrow \log_2(2^{x/7}) = \log_2\left(\frac{16}{3}\right) \Rightarrow \frac{x}{7} = \log_2\left(\frac{16}{3}\right) \Rightarrow x = 7 \log_2\left(\frac{16}{3}\right) (\approx 16.905)$.

Problem 4.

- (a) $3^2 10^{2 \log 5} = 9(10^{\log 25}) = 9(25) = 225$.
- (b) $5e^{-3 \ln 2} = 5e^{\ln 2^{-3}} = 5(2^{-3}) = \frac{5}{8}$.

Problem 5.

- (a) $10^{\log 2+1} = 10^{\log 2} 10^1 = 2(10) = 20$.
- (b) $e^{3-\ln 2} = e^3 e^{-\ln 2} = e^3 e^{\ln 2^{-1}} = e^3(2^{-1}) = \frac{e^3}{2}$.

Problem 7.

- (a) $10^{\log 2 - \log 3} = 10^{\log 2} 10^{-\log 3} = 2(10^{\log 3^{-1}}) = 2(3^{-1}) = \frac{2}{3}$.
- (b) $e^{2 \ln 5 - \ln 2} = e^{2 \ln 5} e^{-\ln 2} = e^{\ln 5^2} e^{\ln 2^{-1}} = (5^2)(2^{-1}) = \frac{25}{2}$.

Problem 5.

- (a) $2^{x^2+x} = 3^x \Rightarrow \ln(2^{x^2+x}) = \ln(3^x) \Rightarrow (x^2+x)\ln 2 = x\ln 3 \Rightarrow (\ln 2)x^2 + (\ln 2 - \ln 3)x = 0 \Rightarrow x(x\ln 2 + \ln 2 - \ln 3) = 0 \Rightarrow x = 0$ or $x = \frac{\ln 3}{\ln 2} - 1$.
- (b) $\log_3(3^{x^2+2x}) = \log_3 1 \Rightarrow x^2 + 2x = 0 \Rightarrow x(x+2) = 0 \Rightarrow x = 0$ or $x = -2$.
- (c) $12\ln x - 2(\ln 2 + \ln x) = 10 \Rightarrow 10\ln x = 10 + 2\ln 2 \Rightarrow \ln x = \frac{10+2\ln 2}{10} \Rightarrow x = e^{\frac{10+2\ln 2}{10}}$.
- (d) $(e^x + 3)(e^x - 2) = 0 \Rightarrow e^x = -3$ (not possible) or $e^x = 2 \Rightarrow x = \ln 2$.
- (e) $e^{2x} + 8 = 6e^x \Rightarrow e^{2x} - 6e^x + 8 = 0 \Rightarrow (e^x - 2)(e^x - 4) = 0 \Rightarrow e^x = 4$ or $e^x = 2 \Rightarrow x = \ln 4$ or $x = \ln 2$.
- (f) $(\ln x)(\ln 5) = \ln 4 + \ln x \Rightarrow (\ln x)(\ln 5 - 1) = \ln 4 \Rightarrow \ln x = \frac{\ln 4}{\ln 5 - 1} \Rightarrow x = e^{\frac{\ln 4}{\ln 5 - 1}}$.

Problem 9.

$$\ln(\sqrt{\pi}3^{1+2x}) = \ln(5^x) \Rightarrow \ln(\sqrt{\pi}) + \ln(3^{1+2x}) = x\ln(5) \Rightarrow \ln(\sqrt{\pi}) + (1+2x)\ln(3) = x\ln(5) \Rightarrow \ln(\sqrt{\pi}) + \ln(3) = x(\ln(5) - 2\ln(3)) \Rightarrow x = \frac{\ln(\sqrt{\pi}) + \ln(3)}{\ln(5) - 2\ln(3)} (\approx -2.843).$$

Problem 11.

$$\ln(e^{2+x}) = \ln(\pi^{3x+3}) \Rightarrow 2+x = (3x+3)\ln(\pi) \Rightarrow 2-3\ln(\pi) = 3x(\ln(\pi)) - x \Rightarrow 2-3\ln(\pi) = x(3\ln(\pi) - 1) \Rightarrow x = \frac{2-3\ln(\pi)}{3\ln(\pi) - 1} (\approx -0.589).$$

Problem 13.

$$7 + \pi 3^{x+2} = 6\pi \Rightarrow 3^{x+2} = \frac{6\pi-7}{\pi} \Rightarrow \ln(3^{x+2}) = \ln\left(\frac{6\pi-7}{\pi}\right) \Rightarrow (x+2)\ln(3) = \ln(6\pi-7) - \ln(\pi) \Rightarrow x+2 = \frac{\ln(6\pi-7) - \ln(\pi)}{\ln(3)} \Rightarrow x = \frac{\ln(6\pi-7) - \ln(\pi)}{\ln(3)} - 2 (\approx -0.792).$$

Problem 15.

$$\ln x^2 = 3 + \ln x \Rightarrow 2\ln x = 3 + \ln x \Rightarrow \ln x = 3 \Rightarrow x = e^3 (\approx 20.086).$$

Problem 17.

$$[\ln(2x+3)]^2 = 9 \Rightarrow \ln(2x+3) = 3 \text{ or } \ln(2x+3) = -3 \Rightarrow 2x+3 = e^3 \text{ or } 2x+3 = e^{-3} \Rightarrow x = \frac{e^3-3}{2} (\approx 8.543) \text{ or } x = \frac{e^{-3}-3}{2} (\approx -1.475).$$

Problem 19.

$$e^x(e^x - 5) = 0 \Rightarrow e^x = 0 \text{ (not possible) or } e^x - 5 = 0 \Rightarrow e^x = 5 \Rightarrow x = \ln 5 (\approx 1.609).$$

Problem 21.

$$e^{2x} - 4e^x + 3 = 0 \Rightarrow (e^x - 3)(e^x - 1) = 0 \Rightarrow e^x = 3 \text{ or } e^x = 1 \Rightarrow x = \ln 3 (\approx 1.099) \text{ or } x = \ln(1) = 0.$$

Problem 23.

$$e^{-2x} - e^{-x} = 6 \Rightarrow (e^{-x})^2 - e^{-x} - 6 = 0 \Rightarrow (e^{-x} - 3)(e^{-x} + 2) = 0 \Rightarrow e^{-x} = 3 \text{ or } e^{-x} = -2 \text{ (not possible)} \Rightarrow x = -\ln(3) = \ln\left(\frac{1}{3}\right) (\approx -1.099).$$

Problem 27.

$$3^{\ln x} = 5x \Rightarrow \ln(3^{\ln x}) = \ln 5x \Rightarrow (\ln x)(\ln 3) = \ln 5 + \ln x \Rightarrow (\ln x)((\ln 3) - 1) = \ln 5 \Rightarrow \ln x = \frac{\ln 5}{(\ln 3) - 1} \Rightarrow x = e^{\frac{\ln 5}{(\ln 3) - 1}} (\approx 1.225 \times 10^7).$$

Problem 32.

$$\ln(x-3) - \ln(2x+1) = 1 \Rightarrow \ln\left(\frac{x-3}{2x+1}\right) = 1 \Rightarrow \frac{x-3}{2x+1} = e \Rightarrow x-3 = e(2x+1) \Rightarrow x(1-2e) = e+3 \Rightarrow x = \frac{e+3}{1-2e} \approx -1.289. \text{ As values of } x \text{ must be greater than } 3 \text{ in the original equation, there is no solution.}$$