Problem 1.

$$y = x^2 \cdot 2^x \Rightarrow \frac{dy}{dx} = (2x)(2^x) + (\ln(2)2^x)(x^2) = x(2^x)(2 + \ln(2)x).$$

Problem 2.

$$y = \frac{5 \cdot 2^x}{3} \Rightarrow \frac{dy}{dx} = \frac{5}{3}(\ln(2)2^x).$$

Problem 3.

$$y = \frac{x^5 5^x}{5} \Rightarrow \frac{dy}{dx} = \frac{1}{5}((5x^4)(5^x) + (\ln(5)5^x)(x^5)) = \frac{1}{5}(x^4)(5^x)(5 + \ln(5)x).$$

Problem 4.

$f(x) = x \cdot 2^x \Rightarrow f'(x) = 2^x + x(2^x \ln 2) = 2^x(1 + x\ln 2)$. Now $f'(x) = 0 \Rightarrow 1 + x\ln 2 = 0 \Rightarrow x = -\frac{1}{\ln 2}$ is the only critical point. Moreover, note that $f'(x) < 0$ for $x < -\frac{1}{\ln 2}$ and $f'(x) > 0$ for $x > -\frac{1}{\ln 2}$. Hence the absolute minimum value of $f(x)$ occurs at $x = -\frac{1}{\ln 2}$ and is $f(-\frac{1}{\ln(2)}) = (-\frac{1}{\ln(2)})2^{-\frac{1}{\ln(2)}} \approx -0.53$.

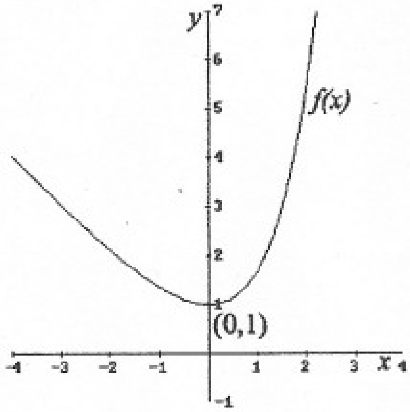
Problem 5.

(a) $f(x) = x^2 + e^x + x^e + e^2 \Rightarrow f'(x) = 2x + e^x + ex^{e-1}$.

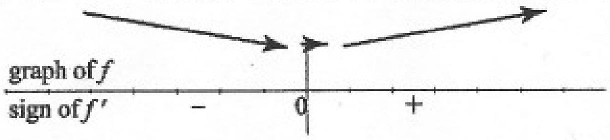
(b) $f(x) = (\pi - \frac{6}{\sqrt{29}})e^x \Rightarrow f'(x) = (\pi - \frac{6}{\sqrt{29}})e^x$

(c) $f(x) = (3e^3)e^x \Rightarrow f'(x) = (3e^3)e^x = 3e^{x+3}$

Problem 8.

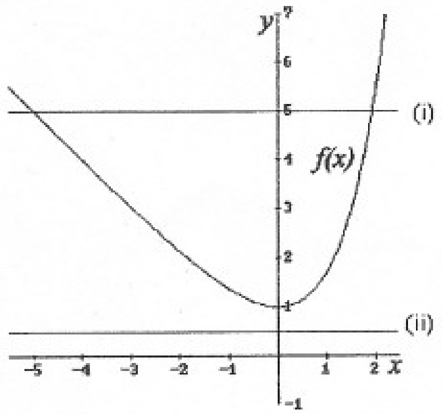


(a) $f'(x) = e^x - 1$. $f'(x) = 0$ when $x = 1$. $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$.



(b) By the first derivative test and part (a), $x = 0$ is the only local minimum point. The corresponding local minimum value is $f(0) = e^0 - 0 = 1$. See the labeled graph above.

(c)



(i) 2 solutions

(ii) No solution

Problem 6.

$f(x) = x\ln\left(\frac{1}{x}\right) = x\ln(x^{-1}) = -x\ln x \Rightarrow f'(x) = -\left((1)\ln x + \left(\frac{1}{x}\right)(x)\right) = -\ln x - 1.$

Problem 7.

$f(x) = \frac{3\ln(3^6x^7)}{\pi} + \frac{3\ln(3^6)}{\pi} = \frac{3\ln(3^6)+3\ln(x^7)}{\pi} + \frac{3\ln(3^6)}{\pi} = \frac{6\ln(3^6)+21\ln x}{\pi} = \Rightarrow f'(x) = \frac{21}{\pi x}.$

Problem 8.

$f(x) = e^{5x}\ln\left(\frac{\pi}{\sqrt{x}}\right) = e^{5x}(\ln\pi - \ln\sqrt{x}) = e^{5x}\left(\ln\pi - \frac{1}{2}\ln x\right) = \left(e^{5x}\right)\left(\ln\pi - \frac{1}{2}\ln x\right)$

$\Rightarrow f'(x) = \left(5e^{5x}\right)\left(\ln\pi - \frac{1}{2}\ln(x)\right) + \left(-\frac{1}{2x}\right)\left(e^{5x}\right) = e^{5x}\left(5\left(\ln\pi - \frac{\ln x}{2}\right) - \frac{1}{2x}\right).$

Problem 9.

$f(x) = 3^x(\log x) = 3^x\left(\frac{\ln x}{\ln 10}\right) = \left(\frac{3^x\ln x}{\ln 10}\right) \Rightarrow f'(x) = \frac{((\ln 3)3^x)(\ln x) + (3^x)\left(\frac{1}{x}\right)}{\ln 10} = \frac{3^x(x(\ln 3)(\ln x) + 1)}{x\ln 10}.$

Problem 10.

$f(x) = \frac{\ln(2x^3)}{3e^x} = \frac{\ln 2 + 3\ln x}{3e^x} \Rightarrow f'(x) = \frac{\left(\frac{3}{x}\right)(3e^x) - (\ln 2 + 3\ln x)(3e^x)}{9e^{2x}} = \frac{3 - x\ln 2 - 3x\ln x}{3xe^x}.$

Problem 18.

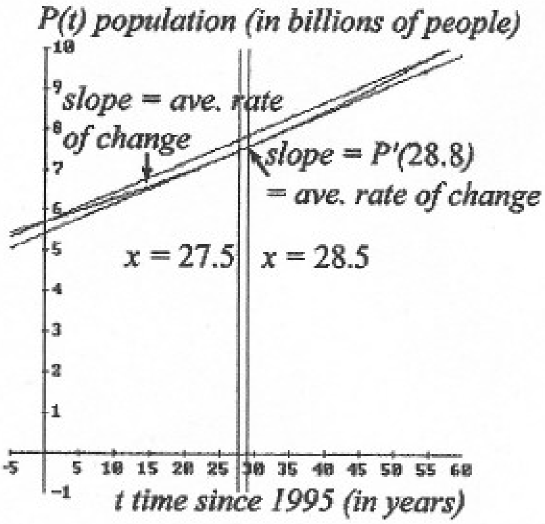
(a) $\frac{9.8 \text{ billion}}{5.7 \text{ billion}} \approx 1.719$. The world's population will increase by 71.9% .

(b) Let $P(t)$ be the total population of the world in billions t years since 1995. As $P(t)$ is exponential, $P(t)$ has the form $P(t) = P_0e^{kt}$, where P_0 and k are constants. Now $P(0) = 5.7 = P_0$, and thus $P(t) = 5.7e^{kt}$. As $P(55) = 9.8$, we have that $P(55) = 9.8 = 5.7e^{55k} \Rightarrow e^{55k} = \frac{9.8}{5.7} \Rightarrow 55k = \ln\left(\frac{9.8}{5.7}\right) \Rightarrow k = \frac{1}{55}\ln\left(\frac{9.8}{5.7}\right) \approx 0.00985$. Therefore, $P(t) = 5.7e^{0.00985t}$. Now $\frac{P(t+1)}{P(t)} = \frac{e^{0.00985(t+1)}}{e^{0.00985t}} = e^{0.00985} \approx 1.0099$. Therefore the annual percentage growth rate is about 0.99%.

(c) $P'(t) = (0.00985)5.7e^{0.00985t} = 0.056145e^{0.00985t} \Rightarrow P'(0) = 0.056145e^{0.00985(0)} = 0.056145$ and $P'(55) = 0.056145e^{0.00985(55)} \approx 0.096513957$. At $t = 0$, the population is growing at a rate of 56.145 million people per year, and at $t = 55$, the population is growing at 96.513957 million people per year.

(d) The average rate of growth is $\frac{P(55)-P(0)}{55-0} = \frac{9.8-5.7}{55} \approx 0.074545454$ billion people per year $= 74,545,455$ people per year.

(e) Solve $\frac{P(55)-P(0)}{55-0} = \frac{4.1}{55} = 0.056145e^{0.00985t}$. Now $e^{0.00985t} = \frac{4.1}{(55)(0.056145)} \Rightarrow 0.00985t = \ln\left(\frac{4.1}{(55)(0.056145)}\right)$ $\Rightarrow t = \left(\frac{1}{0.00985}\right)\ln\left(\frac{4.1}{(55)(0.056145)}\right) \approx 28.8$ years. In 2024, the instantaneous rate of growth and the average rate of growth over the 55-year period are equal.



(f) For North America, $N(t) = N_0(1 + (0.30)\frac{t}{55}) = \frac{0.3N_0t}{55} + N_0$. For Africa, $A(t) = A_0(1 + (3.00)\frac{t}{55}) = \frac{3A_0t}{55} + A_0$.