Due 12/20

16.1

Problem 4.

$f(x) = \frac{x}{(x^3+7x)^4} = \frac{x}{(x^2+7)^4x^4} = \frac{1}{(x^2+7)^4x^3} = (x^2+7)^{-4}x^{-3} \Rightarrow$
$f'(x) = \left(-4(x^2+7)^{-5}\left(\frac{d}{dx}(x^2+7)\right)\right)(x^{-3}) + (-3x^{-4})((x^2+7)^{-4}) =$
$(-8x(x^2+7)^{-5})(x^{-3}) + (-3x(x^2+7))^{-4} = -8x^{-2}(x^2+7)^{-5} + (-3x^3-21x)^{-4} = -\frac{8x^2}{(x^2+7)^5} + \frac{1}{(3x^3+21x)^4}.$

Problem 7.

$f(x) = \left(1-\frac{1}{x}\right)e^{-x} \Rightarrow f'(x) = x^{-2}e^{-x} + \left(e^{-x}\left(\frac{d}{dx}(-x)\right)\right)\left(1-\frac{1}{x}\right) =$
$x^{-2}e^{-x} + (-e^{-x})\left(1-\frac{1}{x}\right) = e^{-x}\left(x^{-2}+\frac{1}{x}-1\right) = \frac{e^{-x}(x^2-x-1)}{x^2}.$

Problem 10.

$f(x) = (3x^3+2x)^{13} \Rightarrow f'(x) = 13(3x^3+2x)^{12}\left(\frac{d}{dx}(3x^3+2x)\right) = 13(3x^3+2x)^{12}(9x^2+2).$

Problem 12.

$f(x) = \frac{\pi^2}{3(x^3+2)^6} = \frac{\pi^2}{3}(x^3+2)^{-6} \Rightarrow f'(x) = \frac{\pi^2}{3}\left(-6(x^3+2)^{-7}\left(\frac{d}{dx}(x^3+2)\right)\right) =$
$-2\pi^2(x^3+2)^{-7}(3x^2) = -6\pi^2x^2(x^3+2)^{-7}.$

Problem 16.

$f(x) = \frac{4}{\sqrt{e^x+1}} = 4(e^x+1)^{-1/2} \Rightarrow f'(x) = 4\left(-\frac{1}{2}\right)(e^x+1)^{-3/2}\left(\frac{d}{dx}(e^x+1)\right) =$
$-2e^x(e^x+1)^{-3/2}.$

Problem 19.

$f(x) = \ln(e^x+x^2) \Rightarrow f'(x) = \frac{1}{e^x+x^2}\left(\frac{d}{dx}(e^x+x^2)\right) = \frac{e^x+2x}{e^x+x^2}.$

Problem 20.

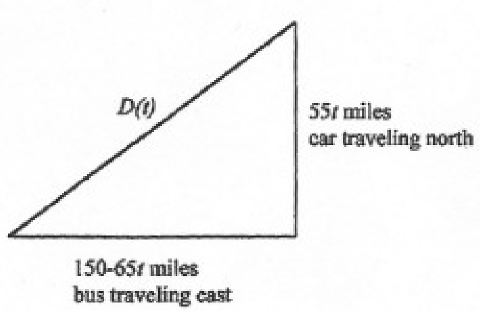
$f(x) = x\ln\left(\frac{x}{x^2+1}\right) = x(\ln(x)-\ln(x^2+1)) \Rightarrow$
$f'(x) = (1)(\ln(x)-\ln(x^2+1)) + (x)\left(\frac{1}{x} - \frac{1}{x^2+1}\left(\frac{d}{dx}(x^2+1)\right)\right) = \ln(x) - \ln(x^2+1) + 1 - \frac{x}{x^2+1}(2x) =$
$\ln(x) - \ln(x^2+1) + 1 - \frac{2x^2}{x^2+1}.$

16.3

Problem 1.

Let $D(t)$ be the distance between the vehicles at time t. We can just minimize $S(t) = (D(t))^2$ increases as $D(t)$ increases for $D(t) \geq 0$, and hence $S(t)$ is minimized at the same value of t for which $D(t)$ is minimized.

(a) From the Pythagorean Theorem see that $S(t) = (D(t))^2 = (55t)^2 + (150-65t)^2$.



Now $S'(t) = 2(55t)(55) + 2(150-65t)(-65) = 14,500t - 19,500$, and $S'(t) = 0$ when $t = \frac{39}{29} \approx 1.345$. As $S(t)$ is a quadratic polynomial with positive coefficient of t^2, $S(t)$ is minimized at $t = \frac{39}{29}$. Therefore the bus and the car will be closest about 1.345 hours after noon.

(b) The minimum distance is $D\left(\frac{39}{29}\right) = \sqrt{\left(55(\left(\frac{39}{29}\right)\right)^2 + \left(150 - 65\left(\frac{39}{29}\right)\right)^2} \approx 96.9$ miles.

(c) The bus is $150 - 65\left(\frac{39}{29}\right) \approx 62.6$ miles from the intersection.