Due 1/10/05

116.2

Problem 1.
$y' = 2\pi x^{2\pi-1} + 2\ln(\pi)\pi^x$.

Problem 2.
$y' = \sqrt{3}(2x^2+1)^{\sqrt{3}-1}(4x) = 4\sqrt{3}x(2x^2+1)^{\sqrt{3}-1}$.

Problem 3.
$y' = 3(\sqrt{2}+1)(3x)^{\sqrt{2}} - \frac{1}{2\sqrt{\pi x^3}}$.

Problem 4.

(a) $y' = \pi e^{3t^2+\pi}(6t) = 6\pi t e^{3t^2+\pi}$.

(b) $y' = \frac{1}{e^t+1}(e^t) = \frac{e^t}{e^t+1}$.

(c) $y = \frac{\pi^2}{\sqrt{x^2+4}} = \pi^2(x^2+4)^{-1/2} \Rightarrow$
$y' = -\pi^2\left(\frac{1}{2}\right)(x^2+4)^{-3/2}(2x) = -\pi^2 x(x^2+4)^{-3/2} = -\frac{\pi^2 x}{(x^2+4)^{3/2}}$.

(d) $y = \frac{1}{(\ln x)^{2.6}} = (\ln x)^{-2.6} \Rightarrow y' = -2.6(\ln x)^{-3.6}\left(\frac{1}{x}\right) = -\frac{2.6}{x(\ln(x))^{3.6}}$.

(e) $y = \frac{1}{(\ln x^2)^{1.5}} = (2\ln x)^{-1.5} \Rightarrow y' = -1.5(2\ln x)^{-2.5}\left(\frac{2}{x}\right) = -\frac{3}{x(2\ln(x))^{2.5}}$.

(f) $y' = \frac{1}{3}(\ln(e^t+1))^{-2/3}(\frac{1}{e^t+1}(e^t)) = \frac{e^t}{(3e^t+3)(\ln(e^t+1))^{2/3}}$.

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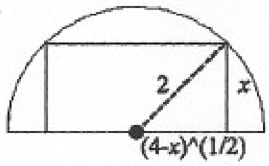
Problem 2.
Let x be the length of a vertical side of the rectangle as shown in the figure.



From the Pythagorean Theorem, we have that half the length of a horizontal side of the rectangle is $\sqrt{4-x^2}$. In terms of x, the area of the rectangle is $A(x) = 2x\sqrt{4-x^2}$. We now maximize $A(x)$ on the interval $[0, 2]$. We have

$$A'(x) = 2\sqrt{4-x^2} + 2x\left(\frac{-2x}{2\sqrt{4-x^2}}\right) = \frac{2(4-x^2)-2x^2}{\sqrt{4-x^2}} = \frac{4(2-x^2)}{\sqrt{4-x^2}}.$$

On $(0, 2)$, $A'(x) = 0$ when $x = \sqrt{2}$, and $A'(x) < 0$ on $(\sqrt{2}, 2)$ and $A'(x) > 0$ on $(0, \sqrt{2})$. Hence the area is maximized when $x = \sqrt{2}$. The dimensions of the rectangle of largest area are $2\sqrt{2} \times \sqrt{2}$, and the area of this rectangle is 4.

Problem 8.

(a) $\left.\frac{dy}{dx}\right|_{x=0} = f'(g(0))g'(0) = f'(2)(-1) = 2.5(-1) = 2.5$.

(b) $y(0) = g(f(0)) = g(3) = 1$.

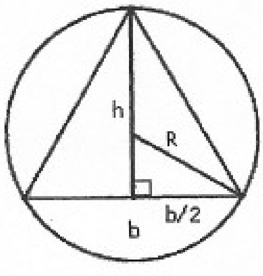
(c) $\frac{d}{dx}(f(x)(g(x))^{-1})\big|_{x=3} = (f'(3)(g(3))^{-1} - ((g(3))^{-2}g'(3)f(3)) = 5(1) - (1)^2(4)(3) = -7$.

(d) $\frac{dy}{dx} = 2f(x)f'(x)g(x^2) + [f(x)]^2 g'(x^2)(2x)$. $\left.\frac{dy}{dx}\right|_{x=2} = 2f(2)f'(2)g(4) + [f(2)]^2 g'(4)(4) = 2(0.3)(2.5)(3) + (0.3)^2(5)(4) = 6.3$.

(e) $y'(x) = \frac{1}{2}(f(x^2))^{-1/2}f'(x^2)(2x) = \frac{xf'(x^2)}{\sqrt{f(x^2)}}$. $y'(2) = \frac{4}{\sqrt{11}} = \frac{4}{\sqrt{11}} = \frac{4\sqrt{11}}{11}$.

Problem 31.

Let b be the base and h the height of the inscribed triangle.



From the figure and Pythagorean Theorem, we have that $\frac{1}{2}b = \sqrt{R^2-(h-R)^2} = \sqrt{2hR-h^2}$. The area of the triangle is $A(h) = \frac{1}{2}bh = h\sqrt{2hR-h^2}$. We now maximize $A(h)$ on the interval $[0, 2R]$. Differentiating, we obtain $A'(h) = (1)(\sqrt{2hR-h^2}) + (\frac{1}{2}(2hR-h^2)^{-1/2}(2R-2h))(h) =$
$\sqrt{2hR-h^2} + (2hR-h^2)^{-\frac{1}{2}}h(R-h)$
$A'(h) = 0$ when:
$\sqrt{2hR-h^2} = -(2hR-h^2)^{-\frac{1}{2}}h(R-h)$
$2hR - h^2 = -h(R-h)$
$3hR = 2h^2$
$h = \frac{3}{2}R$ or $h = 0$.
To maximize $A(h)$ we compute its value at its critical points $h = 0, \frac{3}{2}R$, and $2R$.

h	$A(h)$
0	0
$\frac{3}{2}R$	$\frac{3\sqrt{3}}{4}R^2$
$2R$	0

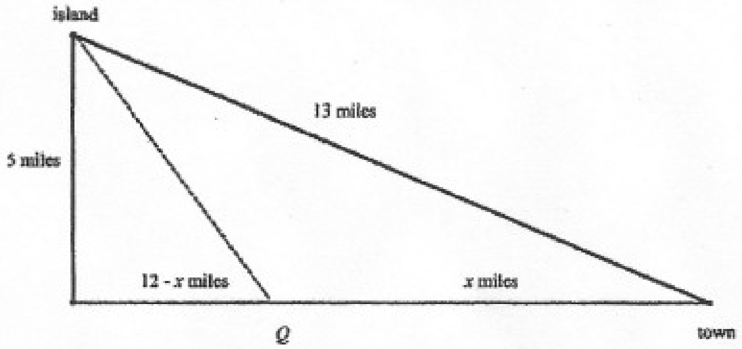
Hence $A(h)$ is maximized when $h = \frac{3}{2}R$. Now the length of the base is $b = 2\sqrt{3R^2 - \frac{9}{4}R^2} = \sqrt{3}R$. By the Pythagorean Theorem, we compute the length of one of the two equal-length sides to be

$$L = \sqrt{\frac{b^2}{4} + h^2} = \sqrt{\frac{3R^2}{4} + \frac{9R^2}{4}} = \sqrt{3}R.$$

As all sides of triangle have equal lengths, the triangle is equilateral.

Problem 34.

Let c be the cost in dollars per mile of cable under ground. The cost of the installing the cable under water is $1.6c$ dollars per mile.



From the Pythagorean Theorem and the figure above, we see that the distance from the point P closest to the island on the shore is $\sqrt{13^2 - 5^2} = 12$ miles. Suppose the cable is run from the island to a point Q on shore x miles from town. Also from the Pythagorean Theorem and the figure above, we see that the distance the cable runs underwater is $d_w(x) = \sqrt{5^2 + (12-x)^2} = \sqrt{x^2 - 24x + 169}$. The total cost of installing the cable is $C(x) = cx + 1.6cd_w(x) = cx + 1.6c\sqrt{x^2 - 24x + 169}$. We now minimize $C(x)$ on $[0, 12]$. Differentiating, we obtain

$$C'(x) = c + 1.6c(\frac{1}{2}(x^2 - 24x + 169)^{-\frac{1}{2}}(2x - 24)) = c + 1.6c(x^2 - 24x + 169)^{-\frac{1}{2}}(x - 12) = \frac{c(5\sqrt{x^2 - 24x + 169} - 8(x - 12))}{5\sqrt{x^2 - 24x + 169}}.$$

Now $C'(x) = 0$ when $5\sqrt{x^2 - 24x + 169} - 8(x - 12) = 0$. We now solve this last equation for x to determine the critical points of $C(x)$ on $(0, 12)$.

$$5\sqrt{x^2 - 24x + 169} = 8(x - 12)$$
$$\sqrt{x^2 - 24x + 169} = 1.6(x - 12)$$
$$x^2 - 24x + 169 = 1.6^2(144 - 24x + x^2)$$
$$1.56x^2 - 37.44x + 199.64 = 0$$

Using the quadratic formula, we obtain $x = \frac{37.44 \pm \sqrt{(-37.44)^2 - 4(1.56)(199.64)}}{2(1.56)} = 12 \pm \frac{25\sqrt{39}}{39}$. Decimal approximations of these values are $x = 16.0032$ and $x = 7.9968$. As the domain of $C(x)$ is $[0, 12]$, the only critical points in $(0, 12)$ is $x = 25 - \frac{25\sqrt{39}}{39} \approx 7.9968$.

To minimize $C(x)$, we compute $C(x)$ at the endpoints $x = 0$ and $x = 12$ and at the critical point $x = 25 - \frac{25\sqrt{39}}{39} \approx 7.9968$

x	$C(x)$
0	20.8c
$25 - \frac{25\sqrt{39}}{39}$	$(12 + \sqrt{39})c \approx 18.245c$
12	20c

Therefore, the cable should run $25 - \frac{25\sqrt{39}}{39} \approx 7.9968$ miles along the shore.