Problem Set # 6

10/8/04

2.3, # 3, 5, 6

2.4, # 4, 6, 10, 12

3.1, # 2

3.2, # 1

Section 2.3

Problem 3.

(a) $f(c), f(d), f(b), f(a)$

(b) $f(b)-f(a)$ negative, $f(b)-f(c)$ positive, $f(d)-f(c)$ positive. $f(b)-f(a) < f(d)-f(c) < f(b)-f(c)$.

(c) $\frac{f(b)-f(a)}{b-a}$ negative, $\frac{f(a)-f(b)}{a-b}$ negative, $\frac{f(c)-f(b)}{c-b}$ negative, $\frac{f(d)-f(c)}{d-c}$ positive, $\frac{f(d)-f(b)}{d-b}$ negative,
$\frac{f(d)-f(b)}{d-b} < \frac{f(c)-f(b)}{c-b} < \frac{f(b)-f(a)}{b-a} = \frac{f(a)-f(b)}{a-b} < \frac{f(d)-f(c)}{d-c}$

Problem 5.

(a) $D(t)$ is positive and decreasing.

(b) The average rate of change from $t=0$ to $t=1$ is $\frac{36,792-149,351}{1-0} = -112,559$ deaths per year.
The percent change is $100\left(\frac{-112,559\,\text{deaths}}{149,351\,\text{deaths}}\right)\% \approx 75.4\%$.

(c) The average rate of change from $t=1$ to $t=2$ is $\frac{36,792-21,222}{2-1} = -15,570$ deaths/year.
The percent change is $100\left(\frac{-15,570\,\text{deaths}}{36,792\,\text{deaths}}\right)\% \approx -42.3\%$.

(d) The average rate of change from $t=2$ to $t=3$ is $[2,3]$ is $\frac{21,222-17,047}{3-2} = -4175$ deaths/year.
The percent change is $100\left(\frac{-4175\,\text{deaths}}{21,222\,\text{deaths}}\right)\% \approx -19.7\%$.

Problem 6.

(a) $t_1 \le t \le t_2$. $(t_1, P(t_1)), (t_2, P(t_2))$. So, the average rate of change is $\frac{P(t_2)-P(t_1)}{t_2-t_1}$

(b) $t_2 \le t \le t_3$. $(t_2, P(t_2)), (t_3, P(t_3))$. So, the average rate of change is $\frac{P(t_3)-P(t_2)}{t_3-t_2}$

(c) Slope of secant line which passes through $(t_2, P(t_2))$ $(t_3, P(t_3))$is bigger for part b) $\Rightarrow$ expression b) is larger

Section 2.4

Problem 4.

(a) I:$(0,2]$; II:$[-2,0)\cup(0,2]$; III:$[-2,0)\cup(2,3.5]$; IV:$(-1,0)\cup(1,2.5]$.

(b) I:$[-2,0)$; II: Never; III:$[-3.5,-2)\cup(0,2)$; IV:$[-2.5,-1)\cup(0,1)$.

(c) I: Graph D; II: Graph C; III: Graph A; IV: Graph B.

Problem 6.

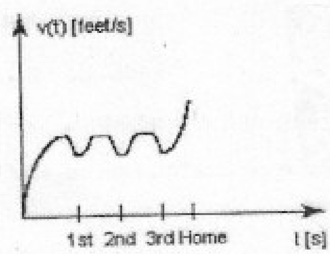
(a) picture d

(b) picture f

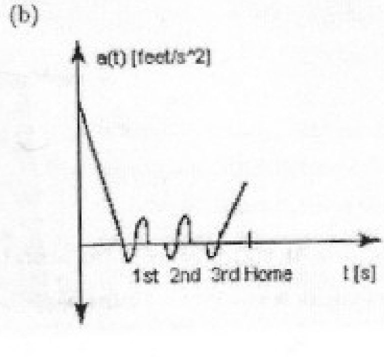
(c) picture e

Problem 10.

(a)



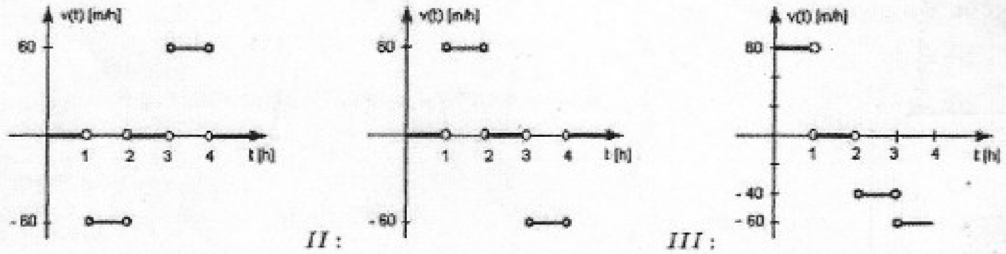
(b)



Problem 12.

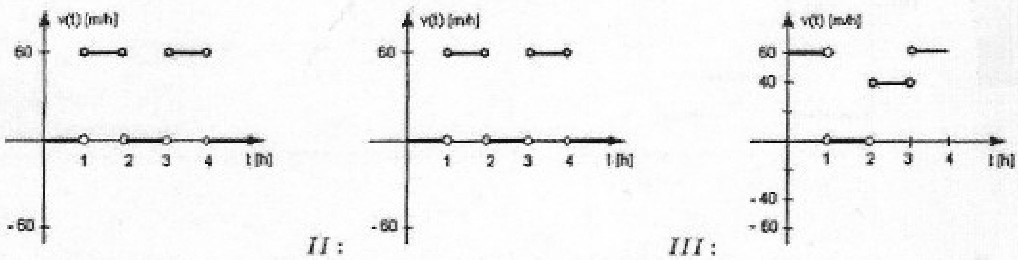
(a) I: We begin 60 miles east of Sturbridge and stay for an hour. Then we travel at 60 mph west to Sturbridge, where we remain for an hour. Then we return to our original position by travelling 60 mph east for an hour, and stay for an hour.
II: We begin in Sturbridge, where we stay for an hour. Then we travel at 60 mph east for one hour, stay for an hour, and return to Sturbridge by travelling west at 60 mph for one hour.
III: We begin 20 miles east of Sturbridge travelling east at 60 mph for one hour. We stay for an hour, and then travel west at 40 mph for one hour, and then at 60 mph for one hour, ending 20 miles west of Sturbridge.

(b)



I: II: III:

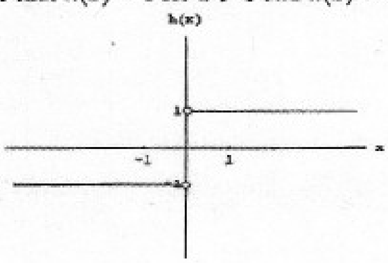
(c)



I: II: III:

Section 3.1

Problem 2.
The function $h(x) = f(x)g(x) = |x|\left(\frac{1}{x}\right) = \frac{|x|}{x}$ is undefined at $x = 0$ because $g(x)$ is undefined at $x = 0$. Note that $h(x) = 1$ for $x > 0$ and $h(x) = -1$ for $x < 0$.



Section 3.2

Problem 1.

(a) $f(g(x)) = \left(\frac{1}{x}\right)^2 = \frac{1}{x^2}$

(b) $g(f(x)) = \frac{1}{x^2}$

(c) $xh(f(x)) = x\left(3(x^2) + 1\right) = 3x^3 + x$

(d) $f(h(g(x))) = \left(3\left(\frac{1}{x}\right) + 1\right)^2 = \left(\frac{3}{x} + 1\right)^2 = \frac{9}{x^2} + \frac{6}{x} + 1$

(e) $g(g(w)) = \frac{1}{\frac{1}{w}} = w$

(f) $h(h(t)) = 3(3t+1) + 1 = 9t + 4$

(g) $g(f(\frac{1}{x})) = \frac{1}{(\frac{1}{x})^2} = \frac{1}{\frac{1}{x^2}} = x^2$

(h) $g(2h(x-1)) = \frac{1}{2(3(x-1)+1)} = \frac{1}{2(3x-2)} = \frac{1}{6x-4}$

(i) As calculated in part (e), $g(g(x)) = x$. We also see that $[g(x)]^2 = \left(\frac{1}{x}\right)^2 = \frac{1}{x^2}$. Now $g(g(-1)) = -1 \neq 1 = \frac{1}{(-1)^2} = [g(-1)]^2$. Therefore, $g(g(x)) \neq [g(x)]^2$.

(j) Note that $[h(x)]^2 = (3x+1)^2 = 9x^2 + 6x + 1$ and $h(x^2) = 3x^2 + 1$. Now $[h(1)]^2 = 16 \neq 4 = h(1) = h(1^2)$. Therefore, $[h(x)]^2 \neq h(x^2)$.