Problem set # 7   10/13/04

3.1 # 6, 9
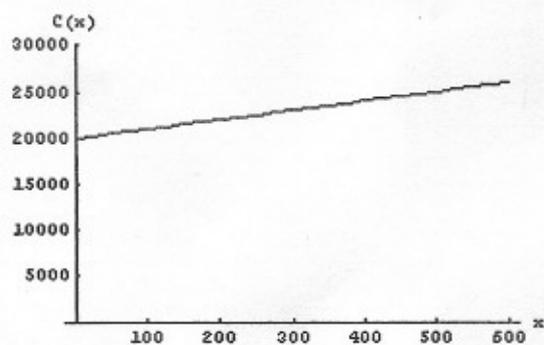3.2 # 3, 4, 14, 15, 47, 48
3.4 # 4

Problem 6.

(a) At the rate of $1500\frac{gal}{h}$. The water flows in at the rate of $500\frac{gall}{h}$. Decreasing at the rate of $1000\frac{gall}{h}$.

(b) At 10:00 A.M. and 6:00 P.M.

(c) Between 10:00 A.M. and 4:00 P.M.
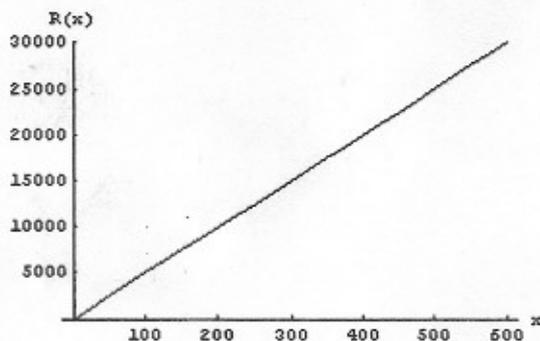
(d) Between 2:00 P.M and 4:00 P.M.

Problem 9.

(a) $C(x) = 20,000 + 10x$.

(b) The total cost is increasing by $\frac{\Delta C}{\Delta x} = \frac{C(x+\Delta x)-C(x)}{x+\Delta x - x} = \frac{(20,000+10(x+\Delta x))-(20,000+10x)}{\Delta x} = 10$ dollars per widget.

(c) $R(x) = 50x$.

(d) $P(x) = R(x) - C(x) = (50x) - (20,000 + 10x) = 40x - 20,000$.

Problem 3.

(a) $g(f(1)) = g(0) = -3$

(b) $f(g(0)) = f(-3) = 0$ (Note: $g(f(0)) = g(1) = -2$)

Problem 4.

(a) $f(f(2)) = f(-2) = 3$

(b) $f(f(1)) = f(0) = 1$

Problem 14.

(a) $f(x) + g(x) = (x - 3) + (x^2 - 6x) = x^2 - 5x - 3$

(b) $f(x) - g(x) = (x - 3) - (x^2 - 6x) = -x^2 + 7x - 3$

(c) $f(x)g(x) = (x - 3)(x^2 - 6x) = x^3 - 6x^2 - 3x^2 + 18x = x^3 - 9x^2 + 18x$

(d) $f(g(x)) = f(x^2 - 6x) = x^2 - 6x - 3$

(e) $g(f(x)) = g(x - 3) = (x - 3)^2 - 6(x - 3) = x^2 - 6x + 9 - 6x + 18 = x^2 - 12x + 27$

(f) $\frac{f(x)}{g(x)} = \frac{x-3}{x^2-6x}$

Problem 15.

(a) x-intercepts of $f(x)$ can be found by solving $f(x) = 0$. So, $x - 3 = 0 \Rightarrow x = 3$ is x-intercept.

For y-intercepts we set $x = 0$, so $y = 0 - 3 = -3$ is y-intercept.

(b) x-intercepts of $g(x)$: $x^2 - 6 = 0$ $x(x - 6) = 0$, $\Rightarrow x = 0$ or $x = 6$.

For y-intercepts we set $x = 0$, so $g(0) = 0^2 - 0 \cdot 6 = 0$

(c) x-intercepts of $f(x) \cdot g(x)$: $x^3 - 9x^2 + 18x = 0$, $x(x^2 - 9x + 18) = 0$, $x(x - 3)(x - 6) = 0 \Rightarrow x = 0$ or $x = 3$ or $x = 6$.

For y-intercepts we set $x = 0$, so $f(0)g(0) = 0^3 - 9 \cdot 0^2 + 18 \cdot 0 = 0$ is y-intercept.

(d) $\frac{f(x)}{g(x)} = \frac{x-3}{x^2-6x}$, $D : x \neq 0$ and $x \neq 6$. x-intercepts: $\frac{x-3}{x^2-6x} = 0 \Leftrightarrow x - 3 = 0$, so $x = 3$ is x-intercept.

For y-intercepts we set $x = 0$, but 0 is excluded from the Domain, so there are no y-intercepts.

Problem 47.

(a) $f(f(2)) = f\left(\frac{1}{2} + 2\right) = f\left(\frac{5}{2}\right) = \frac{2}{5} + \frac{5}{2} = \frac{4}{10} + \frac{25}{10} = \frac{29}{10}$

(b) $g(g(-1)) = g\left(\frac{2(-1)}{(-1)^2+1}\right) = g(-1) = -1$

Problem 48.

(a) $f(g(x)) = f(\frac{2x}{x^2+1}) = \frac{x^2+1}{2x} + \frac{2x}{x^2+1}$

(b) $g(f(x)) = g(\frac{1}{x} + x) = g(\frac{x+1}{x}) = \frac{2 \cdot \frac{x+1}{x}}{(\frac{x+1}{x})^2+1}$

Problem 4.

(a) (iii)

(b) (vi)

(c) (ii)

(d) (i)

(e) (v)