Problem 14.

$f(x) = x^3 + x^2 + x + 1 \Rightarrow f'(x) = 3x^2 + 2x + 1 = 0$. Now $0 = f'(x) \Rightarrow x = \frac{-2\pm\sqrt{-8}}{6}$. Hence there are no real values of x for which $0 = f'(x)$ and consequently no critical points.

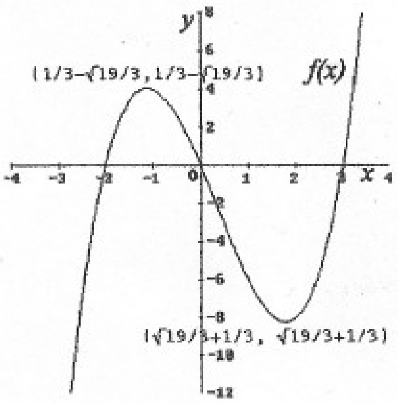
Problem 15.

$f(x) = -2x^3 + x^2 + 7 \Rightarrow f'(x) = -6x^2 + 2x = -2x(3x - 1)$. Now $0 = f'(x) \Rightarrow x = 0$ or $x = \frac{1}{3}$. We compute $f''(x) = -12x + 2$ and see that $f''(0) = 2 > 0$ and that $f''\left(\frac{1}{3}\right) = -2 < 0$. Hence there is a local minimum at $x = 0$ and a local maximum at $x = \frac{1}{3}$. As $\lim_{x\to\infty} f(x) = -\infty$ and $\lim_{x\to-\infty} f(x) = \infty$, neither of the local extrema are absolute extrema.

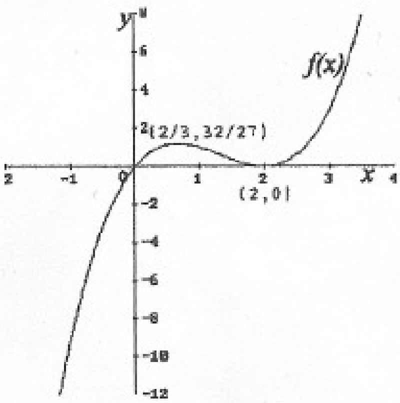
Problem 16.

$f(x) = x^3 + 2x^2 + 3x + 4 \Rightarrow f'(x) = 3x^2 + 4x + 3$. Now $0 = f'(x) \Rightarrow x = \frac{-4\pm\sqrt{4^2-4(3)(3)}}{6}$. Hence there are no real values of x for which $0 = f'(x)$ and consequently no critical points.

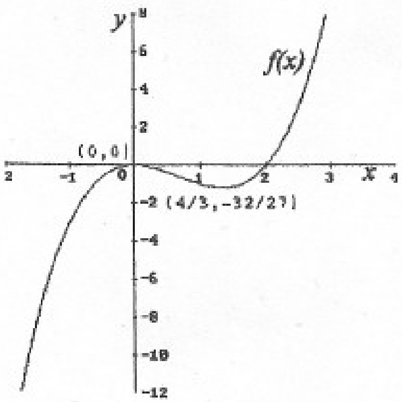
Problem 17.



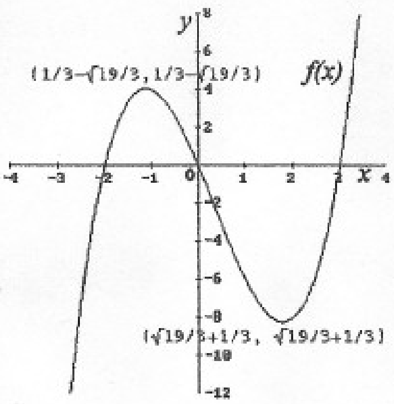
Problem 18.



Problem 19.



Problem 20.



Problem 21.

$f(x) = -2x^3 + 3x^2 + 6x - 2 \Rightarrow f'(x) = -6x^2 + 6x + 6 \Rightarrow f''(x) = -12x + 6 \Rightarrow$ The point of inflection is $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$. Now $f'\left(\frac{1}{2}\right) = \frac{15}{2}$. The equation of the tangent line is $\left(y - \frac{3}{2}\right) = \frac{15}{2}\left(x - \frac{1}{2}\right) \Leftrightarrow y = \frac{15}{2}x - \frac{9}{4}$.

11.2

Problem 10.

$P(x) = kx^2(x-\pi)^2(x+2)$, for some $k \neq 0$. As $\lim_{x\to\infty} P(x) = \infty$, k can be any positive number, and hence the answer is not unique.

Problem 11.

$P(x) = k(x+1)^3$, for some $k \neq 0$. As $\lim_{x\to\infty} P(x) = \infty$, k can be any positive number, and hence the answer is not unique.

Problem 12.

$P(x) = a(x-\pi)^2 + 2$, for some $a \neq 0$. As $0 = P(0) = a(0-\pi)^2 + 2$, $a = -\frac{2}{\pi^2}$. This answer is unique.

Problem 14.

$P(x) = k(x-1)^2(2-x)$, for some $k \neq 0$. Now $\sqrt{e} = P(0) = k(0-1)^2(2-0) \Rightarrow k = \frac{\sqrt{e}}{2}$. This answer is unique.

Problem 16.

(a) $f(x) = -x^3 - x^2 - 5x = -x(x^2+x+5)$. Now if $x^2+x+5 = 0$, then $x = \frac{-1\pm\sqrt{1-4(5)}}{2}$, which are not real solutions. The only zero is $x = 0$.

(b) $g(x) = 0.5x^4 - 0.5 = 0.5(x^2+1)(x-1)(x+1)$. Now $x^2+1 = 0$ has no real solutions; hence, the zeros are $x = -1$ and $x = 1$.

Problem 19.

$P(x) = x^4 - 2x^3 - 6x^2 + 12x = x(x-2)(x^2-6) = x(x-2)(x-\sqrt{6})(x+\sqrt{6})$. The zeros are $x = 0, 2, \sqrt{6}$, and $-\sqrt{6}$.

Problem 20.

$g(x) = 3x^3 + 3 = 3(x+1)(x^2-x+1)$. Now, as $(-1)^2 - 4(1)(1) < 0$, $x^2-x+1 = 0$ has no real solutions. Hence, the zero is $x = -1$.

11.3

Problem 8.

The strategy here should be to use the x−intercepts to establish the factors of the appropriate polynomial and then to use the additional point provided to determine the multiplicative factor.

(a) The zeros of the function are $x = -2, 1$, and 2. As the graph has the shape of a graph of a cubic polynomial, the function could have an equation of the form $P(x) = k(x+2)(x-1)(x-2)$, where k is a constant. Because the graph has a y−intercept of 3, we have $3 = P(0) = k(0+2)(0-1)(0-2) \Rightarrow k = \frac{3}{4}$. Therefore, $P(x) = \frac{3}{4}(x+2)(x-1)(x-2)$.

(b) This graph is the reflection of the graph in part (a) in the x−axis. Thus, $P(x) = -\frac{3}{4}(x+2)(x-1)(x-2)$.

(c) This function has an even order zero at $x = -2$ and an odd order zero at $x = 0$. As the graph has the shape of the graph of a cubic polynomial, the function could have an equation of the form $P(x) = kx(x+2)^2$. Because the graph contains the point $(1, 2)$, we have $2 = P(1) = k(1)(1+2)^2 = 9k \Rightarrow k = \frac{2}{9}$. Therefore, $P(x) = \frac{2}{9}x(x+2)^2$

Problem 12.

(a) (i) True.

(ii) False; $P(x) = (x-1)(x-2)(x-3)(x-4)(x-5)$ has 5 zeros.

(iii) False; $P(x) = x^5 + x$ has derivative $P'(x) = x^4 + 1$ which is positive for all x, forcing $P(x)$ to have no turning points.

(iv) True.

(b) Statement (i) is true. Because $P'(\pi) = 0$ and $P''(\pi) > 0$, $P(x)$ has a local minimum at $x = \pi$; because $P(x)$ has degree 5, it cannot have an absolute minimum.

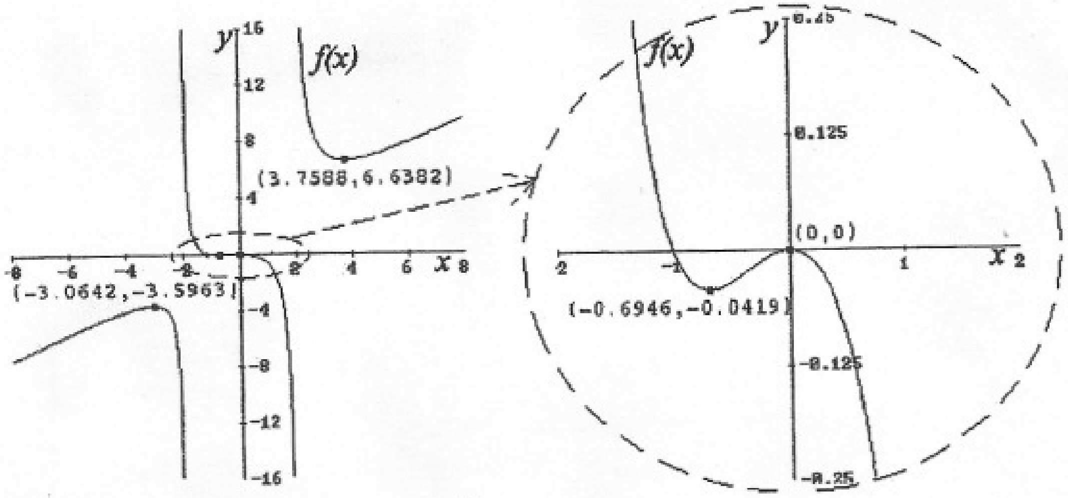
Problem 3.

(a) The graph has no zeros and vertical asymptotes at $x = -1$ and $x == 2$. The sign of y changes across both of these vertical asymptotes, which implies that there are odd powers of $(x+1)$ and $(x-2)$ in the denominator of the function. As there is a horizontal asymptote at $y = 0$, the degree of the numerator of the function is less than the degree of the denominator of the function. Hence the equation has the form $y = \frac{k}{(x+1)(x-2)}$, where k is a nonzero constant. As $y < 0$ for $|x| > 2$, $k < 0$. For simplicity, we choose $k = -1$. Therefore, $y = -\frac{1}{(x+1)(x-2)}$.

(b) The graph has simple zeros at $x = -2$ and $x = 0$ and a vertical asymptote at $x = -1$. As the sign of y does not change across the vertical asymptote, there is an even power of $(x+1)$ in the denominator of the function. As there is a horizontal asymptote at $y = 2$, the degrees of the numerator and denominator of the function are equal, and the lead the coefficient of the numerator is 2. Hence $y = \frac{2x(x+2)}{(x+1)^2}$.

(c) The graph has a simple zero at $x = -2$ and an even-ordered zero at $x = 0$. Thus, the numerator has factors of $(x+2)$ and x^2. There are vertical asymptotes at $x = -1$ and $x = 1$. The sign of y changes across the vertical asymptote $x = 1$ but not across $x = -1$. Thus, the denominator has factors of $(x+1)^2$ and $(x-1)$. As there is a horizontal asymptote at $y = 2$, the degrees of the numerator and denominator of the function are equal, and the lead the coefficient of the numerator is 2. Therefore, the equation has the form $y = \frac{2x^2(x+2)}{(x+1)^2(x-1)}$.

Problem 5.



Calculating the derivative of $f(x) = \frac{x^3+x^2}{x^2-4}$ via the quotient rule gives:

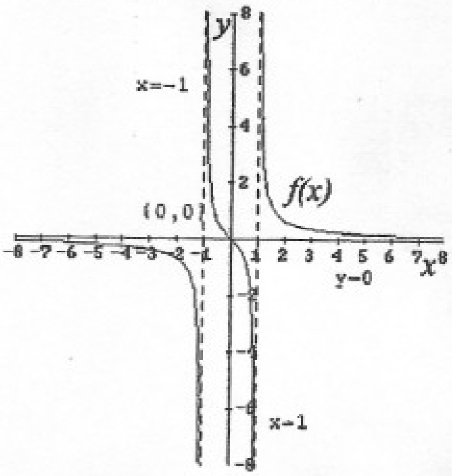
$$f'(x) = \frac{(3x^2+2x)(x^2-4)-(x^3+x^2)(2x)}{(x^2-4)^2} = \frac{(3x^4+2x^3-12x^2-8x)-(2x^4+2x^3)}{(x^2-4)^2} =$$

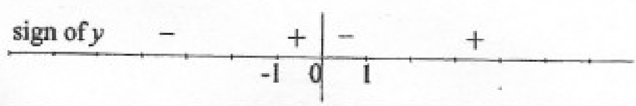
$$\frac{x^4-12x^2-8x}{(x^2-4)^2} = \frac{x(x^3-12x-8)}{(x^2-4)^2} = \frac{x^4-12x^2-8x}{(x^2-4)^2}.$$

Now, as $f'(0) = 0$, $x = 0$ is a critical point. A graphing calculator will verify that there is a local minimum at $x = 0$. Furthermore, a graphing calculator will find a local maximum at $x \approx -3.0642$, a local minimum at $x \approx -0.6946$, and a local minimum at $x \approx 3.7588$.

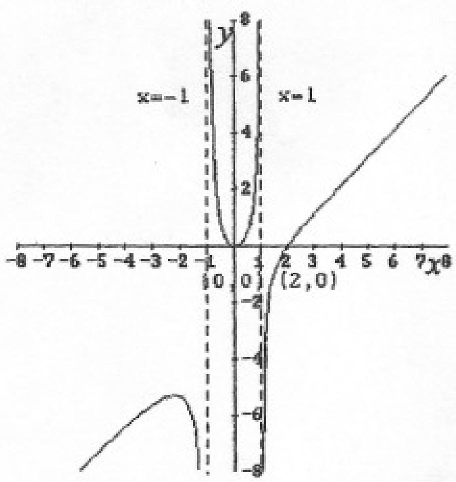
Problem 12.

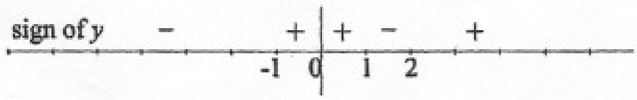
(i) The x-intercept is the origin; the y-intercept is the origin; the vertical asymptotes are $x = \pm 1$; the horizontal asymptote is the $x-$axis.



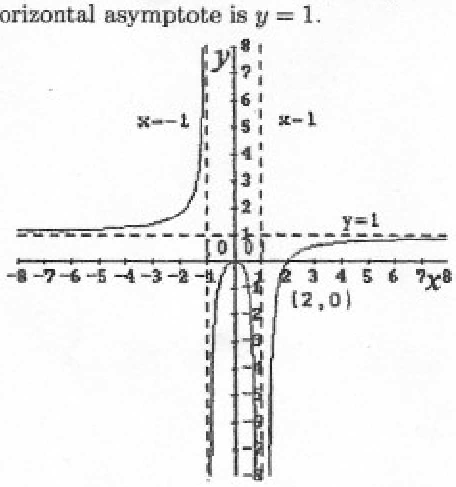


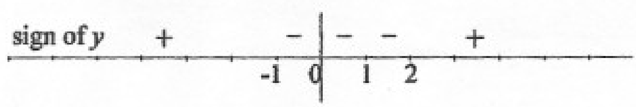
(ii),(iii) The x-intercepts are $x = 0, 2$; the y-intercept is the origin; the vertical asymptotes are $x = \pm 1$; there are no horizontal asymptotes.



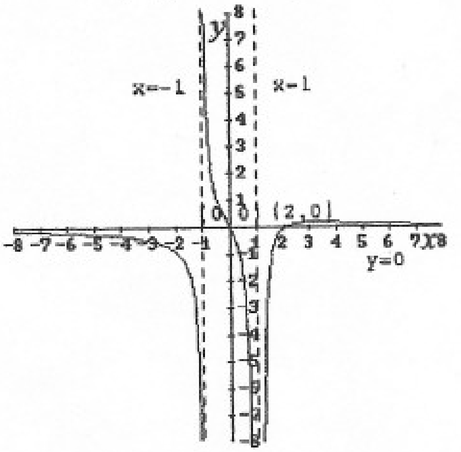


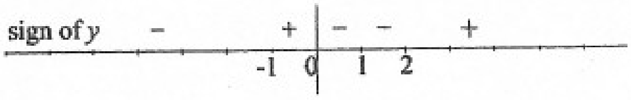
(iv) The x-intercepts are $x = 0, 2$; the y-intercept is the origin; the vertical asymptotes are $x = \pm 1$; the horizontal asymptote is $y = 1$.



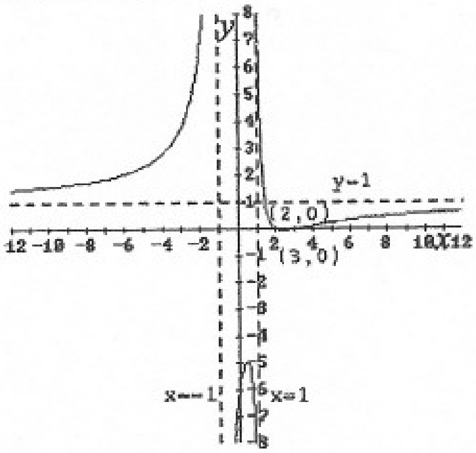


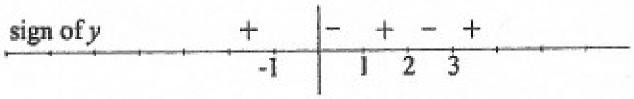
(v) The x-intercepts are $x = 0, 2$; the y-intercept is the origin; the vertical asymptotes are $x = \pm 1$; the horizontal asymptote is the x–axis.



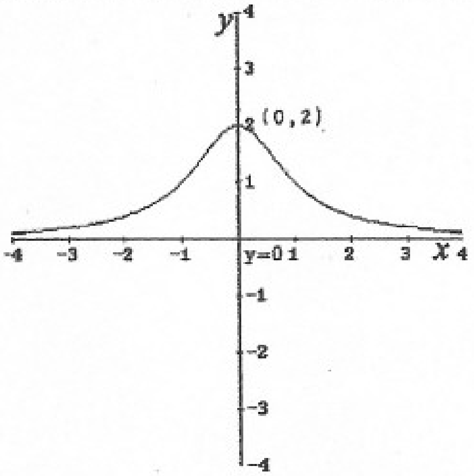


(vi) The x-intercepts are $x = 2, 3$; the y-intercept is $y = -6$; the vertical asymptotes are $x = \pm 1$; the horizontal asymptote is $y = 1$.

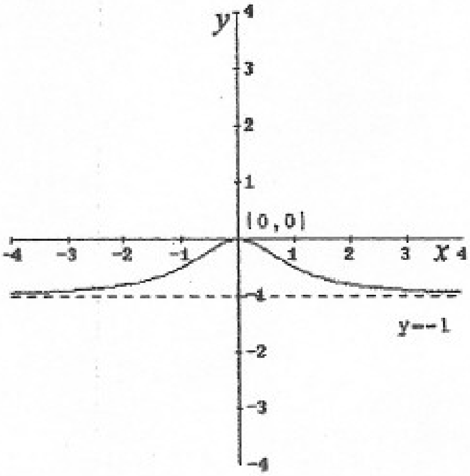




(vii) There are no x-intercepts; the y-intercept is $y = 2$; there are no vertical asymptotes; the horizontal asymptote is the x-axis. Note that $y < 0$ for all values of x.



(viii) The x-intercept is the origin; the y-intercept is the origin; there are no vertical asymptotes; the horizontal asymptote is $y = -1$. Note that $y < 0$ for all values of x.



12.1

Problem 1.

(a) 1-to-1 and invertible (as long as social security numbers are not reassigned after someone dies)

(b) Not 1-to-1 and hence not invertible; five people are put into each group.

(c) Not 1-to-1 and not invertible; many sites have the same altitude.

Problem 2.

(a) $g(x) = \frac{x+3}{6}$; $f(g(x)) = 6\left(\frac{x+3}{6}\right) = g\left(\frac{x+3}{6}\right) - 3 = x+3-3 = x$; $g(f(x)) = g(6x-3) = \frac{(6x-3)+3}{6} = \frac{6x}{6} = x$.

(b) $g(x) = \sqrt[3]{x} + 3$; $f(g(x)) = f(\sqrt[3]{x}+3) = (\sqrt[3]{x}+3-3)^3 = (\sqrt[3]{x})^3 = x$; $g(f(x)) = g((x-3)^2) = \sqrt[3]{(x-3)^3} + 3 = (x-3)+3 = x$.

Problem 8.
(a) False. (b) True. (c) True. (d) True.

12.2

Problem 4.
Given $y = \frac{2x-1}{3x+4}$, interchange x and y and solve for y. Now $x = \frac{2y-1}{3y+4} \Rightarrow 3xy + 4x = 2y - 1 \Rightarrow 3xy - 2y = -4x - 1 \Rightarrow y = \frac{-4x-1}{3x-2}$. Hence $f^{-1}(x) = \frac{-4x-1}{3x+2}$.

Problem 9.
Given $y = 2\sqrt{x-6}$, interchange x and y and solve for y. Now $x = 2\sqrt{y-6} \Rightarrow x^2 = 4(y-6) \Rightarrow y = \frac{1}{4}x^2 + 6$. Hence $f^{-1}(x)\frac{1}{4}x^2 + 6$. The domain of f^{-1} is the range of f, which is $[0, \infty)$.

Problem 10.
Given $y = x^3 + 1$, interchange x and y and solve for y. Now $x = y^3 + 1 \Rightarrow x - 1 = y^3 \Rightarrow y = \sqrt[3]{x-1}$. Hence $f^{-1} = \sqrt[3]{x-1}$. The domain of f^{-1} is $(-\infty, \infty)$.

12.3

Problem 5.

(a) Spending half as much on advertising would result in \$80,000 less in revenue.

(b) Increasing advertising expenditures from \$30,000 to \$30,001 would result in roughly a \$2.80 increase in revenue.

(c) To generate twice as much revenue as last year, the company would need to spend $R^{-1}(2C)$ on advertising.

Problem 6.

(a) $f^{-1}(6) = -\frac{1}{3}$.

(b) Interchange t and y in the equation $y = 5(1.1)^{6t+2} + 1$ and solve for y. Now $t = 5(1.1)^{6y+2} + 1 \Rightarrow \frac{t-1}{5} = (1.1)^{6y+2} \Rightarrow \frac{\ln\left(\frac{t-1}{5}\right)}{\ln 1.1} = 6y + 2 \Rightarrow y = \frac{1}{6}\left(\frac{\ln\left(\frac{t-1}{5}\right)}{\ln 1.1} - 2\right)$. Hence $f^{-1}(t) = \frac{1}{6}\left(\frac{\ln\left(\frac{t-1}{5}\right)}{\ln 1.1} - 2\right)$.
(Note that logarithmic calculations are discussed in Chapter 13.)

(c) Hence $f^{-1}(6) = \frac{1}{6}\left(\frac{\ln\left(\frac{6-1}{5}\right)}{\ln 1.1} - 2\right) = \frac{1}{6}(0-2) = -\frac{1}{3}$.

(d) $f^{-1}(30)$ is the number of days after the dump opens that it will take to accumulate 30 pounds of garbage.

13.1/

Problem 3.

(a) It is between 2 and 3, because $\log_7(50)$ is the number to which 7 must be raised to get 50, and $7^2 = 49$ while $7^3 = 343$.

(b) It is between -1 and 0, because $\log_{10}(0.5)$ is the number to which 10 must be raised to get 0.5, and $10^{-1} = 0.1$ while $10^0 = 1$.

Problem 4.

(a) It is between -2 and -1, because $\log_{10}(0.05)$ is the number to which 10 must be raised to get 0.05, and $10^{-2} = 0.01$ while $10^{-1} = 0.1$.

(b) It is between 3 and 4, because $\log_3(29)$ is the number to which 3 must be raised to get 29, and $3^3 = 27$ while $3^4 = 81$.

Problem 5.
Note that $\sqrt{30}$ is between 5 and 6, because $5^2 = 25$ and $6^2 = 36$.

(a) It is between 2 and 3, since $\log_2 \sqrt{30}$ is the number to which 2 must be raised to get $\sqrt{30}$, and $2^2 = 4$ while $2^3 = 8$.

(b) It is between 1 and 2, since $\log_5 \sqrt{30}$ is the number to which 5 must be raised to get $\sqrt{30}$, and $5^1 = 5$ while $5^2 = 25$.

(c) It is between 0 and 1, since $\log_2 \sqrt{30}$ is the number to which 2 must be raised to get $\sqrt{30}$, and $10^0 = 1$ while $10^1 = 10$.

13.2/

Problem 2.

(a) $\log_2(u^2 w) = \log_2(u^2) + \log_2 w = 2\log_2 u + \log_2 w = 2A + B$

(b) $\log_2(u^3/w^2) = \log_2(u^3) - \log_2(w^2) = 3\log_2 u - 2\log_2 w = 3A - 2B$

(c) $\log_2(1/\sqrt{w}) = \log_2 1 - \log_2 \sqrt{w} = 0 - \log_2 w^{\frac{1}{2}} = -\frac{1}{2}\log_2 w = -\frac{1}{2}B$

(d) $\log_2(\frac{2}{\sqrt{uw}}) = \log_2 2 - \log_2(uw)^{\frac{1}{2}} = 1 - \frac{1}{2}(\log_2(uw)) = 1 - \frac{1}{2}(\log_2 u + \log_2 w) = 1 - \frac{1}{2}(A + B)$

Problem 4.

(a) $3^2 10^{2\log 5} = 9(10^{\log 25}) = 9(25) = 225.$

(b) $5e^{-3\ln 2} = 5e^{\ln 2^{-3}} = 5(2^{-3}) = \frac{5}{8}.$

Problem 6.

(a) $2^{\log_2 3+3} = 2^{\log_2 3}2^3 = 3(8) = 24.$

(b) $e^{2\ln A+1} = e^{2\ln A}e^1 = e^{\ln A^2}e = A^2 e.$

Problem 8.

(a) $10^{-\log\frac{1}{10}} = 10^{\log(\frac{1}{10})^{-1}} = 10^{\log 10} = 10.$

(b) $e^{\frac{\ln 3}{2}} = e^{\frac{1}{2}\ln 3} = (e^{\ln 3})^{\frac{1}{2}} = 3^{\frac{1}{2}} = \sqrt{3}.$

Problem 10.
$5\log\frac{2}{3} = 5(\log 2 - \log 3) = 5(a - b).$

Problem 12.

$5\log\sqrt[3]{6} = 5\log(6^{\frac{1}{3}}) = \frac{5}{3}\log 6 = \frac{5}{3}(\log 2 + \log 3) = \frac{5}{3}(a + b).$

Problem 4.

(a) $5 = (\ln x)(\ln(2) - 3) \Rightarrow \frac{5}{\ln(2)-3} = \ln x \Rightarrow x = e^{\frac{5}{\ln(2)-3}}$.

(b) $7^{1+\log x} = 4 \Rightarrow 1 + \log x = \log_7 4 \Rightarrow x = 10^{\log_7(4)-1}$

(c) $K + L = Le^x - Ke^x \Rightarrow K + L = e^x(L - K) \Rightarrow e^x = \frac{K+L}{L-K} \Rightarrow x = \ln\left(\frac{K+L}{L-K}\right)$.

(d) $R = \frac{(Pn)^x}{(1+n)^{nx}} \Rightarrow \log R = \log(\frac{Pn}{(1+n)^n})^x \Rightarrow \log R = x\log(\frac{Pn}{(1+n)^n}) \Rightarrow \log R = x(\log(Pn) - \log((1+n)^n)) \Rightarrow$ $x = \frac{\log R}{\log(Pn) - n\log(1+n)}$.

(e) $3b^x = c^x 3^{2x} \Rightarrow \log_3(3b^x) = \log_3(c^x 3^{2x}) \Rightarrow \log_3 3 + \log_3 b^x = \log_3 c^x + \log_3 3^{2x} \Rightarrow 1 + x\log_3 b = x\log_3 c + 2x$ $\Rightarrow 1 = x(\log_3 c + 2 - \log_3 b) \Rightarrow x = \dfrac{1}{2 + \log_3 \frac{c}{b}}$.

Problem 8.

$\ln\left(\dfrac{3}{2^{x-3}}\right) = \ln(7^{2x+1}) \Rightarrow \ln(3) - \ln(2^{x-3}) = (2x+1)\ln(7) \Rightarrow \ln(3) - (x-3)\ln(2) = (2x)\ln(7) + \ln(7)$ $\Rightarrow \ln(3) + 3\ln(2) - \ln(7) = x(2\ln(7) + \ln(2)) \Rightarrow x = \frac{\ln(3)+3\ln(2)-\ln(7)}{2\ln(7)+\ln(2)} (\approx 0.269)$.

Problem 12.

$3^x = 0$ or $\frac{5}{3^{x+1}} = 0$; no solution, since $3^x > 0$ for all x's and $\frac{5}{3} \cdot \left(\frac{1}{3}\right)^x > 0$ for all x's.

Problem 14.

$\log x - \log(x+1) = \log(\frac{x}{x+1}) = 2 \Rightarrow \frac{x}{x+1} = 10^2 \Rightarrow x = 100(x+1) \Rightarrow x = -\frac{100}{99}$, which cannot be a solution of the original equation because $x < 0$. $\Rightarrow$ No solution.

Problem 16.

$\ln x^{1/2} + \ln x^2 = 1 - 2\ln x \Rightarrow \frac{1}{2}\ln x + 2\ln x = 1 - 2\ln x \Rightarrow \frac{9}{2}\ln x = 1 \Rightarrow \ln x = \frac{2}{9} \Rightarrow x = e^{\frac{2}{9}} (\approx 1.2494)$.

Problem 18.

$[\ln(2x+3)]^2 = 9 \Rightarrow \ln(2x+3) = 3$ or $\ln(2x+3) = -3 \Rightarrow 2x+3 = e^3$ or $2x+3 = e^{-3} \Rightarrow x = \frac{e^3-3}{2} (\approx 8.543)$ or $x = \frac{e^{-3}-3}{2} (\approx -1.475)$.

Problem 20.

$e^x(e^x - 5) = 6 \Rightarrow (e^x)^2 - 5e^x - 6 = 0$
$(e^x - 6)(e^x + 1) = 0$
$e^x = 6$ or $e^x = -1$ (not possible) $\Rightarrow x = \ln 6 (\approx 1.792)$.

Problem 22.

$2e^{2x} + 6 = 11e^x \Rightarrow 2(e^x)^2 - 11e^x + 6 = 0 \Rightarrow e^x = \frac{11\pm\sqrt{73}}{4} \Rightarrow x = \ln\left(\frac{11\pm\sqrt{73}}{4}\right) (\approx -0.488, 1.586)$.

Problem 25.

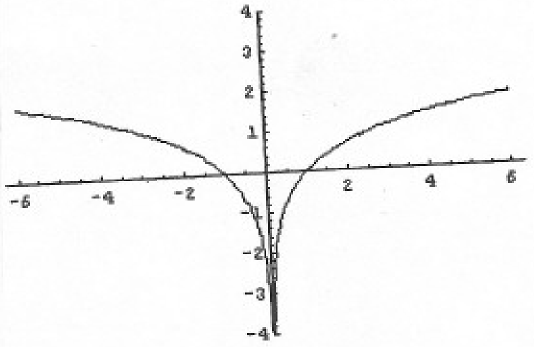
$e^x - 1 = e^{-x} \Rightarrow e^x(e^x - 1) = e^x(e^{-x}) \Rightarrow (e^x)^2 - (e^x) - 1 = 0 \Rightarrow e^x = \frac{1\pm\sqrt{5}}{2}$ (only positive solution is possible) $\Rightarrow e^x = \frac{1+\sqrt{5}}{2} \Rightarrow x = \ln(\frac{1+\sqrt{5}}{2}) (\approx 0.481)$.

Problem 28.

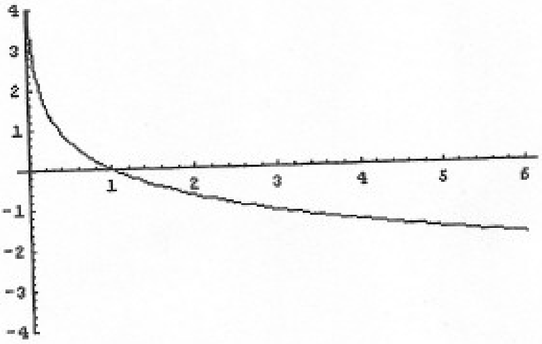
$\frac{4}{\ln(x+1)} + 5 = 13 \Rightarrow \frac{4}{\ln(x+1)} = 8 \Rightarrow 4 = 8\ln(x+1) \Rightarrow \frac{1}{2} = \ln(x+1) \Rightarrow x+1 = e^{\frac{1}{2}} \Rightarrow x = \sqrt{e} - 1 (\approx 0.649)$.

13.4

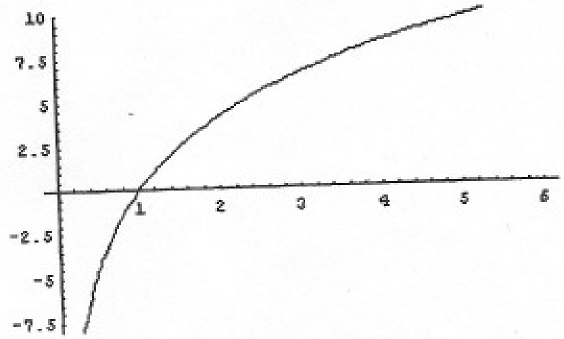
Problem 4.



Problem 5.



Problem 6.



$y = \ln(x) - \ln(x^3) + 4\ln(x^2) =$
$\ln(x) - 3\ln(x) + 8\ln(x) = 6\ln(x)$

14.2

Problem 1.
$y = x^2 \cdot 2^x \Rightarrow \dfrac{dy}{dx} = (2x)(2^x) + (\ln(2)2^x)(x^2) = x(2^x)(2 + \ln(2)x).$

Problem 2.
$y = \frac{5 \cdot 2^x}{3} \Rightarrow \dfrac{dy}{dx} = \dfrac{5}{3}(\ln(2)2^x).$

Problem 3.
$y = \frac{x^5 5^x}{5} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{5}((5x^4)(5^x) + (\ln(5)5^x)(x^5)) = \dfrac{1}{5}(x^4)(5^x)(5 + \ln(5)x).$

Problem 4.
$f(x) = x \cdot 2^x \Rightarrow f'(x) = 2^x + x(2^x \ln 2) = 2^x(1 + x\ln 2)$. Now $f'(x) = 0 \Rightarrow 1 + x\ln 2 = 0 \Rightarrow x = -\frac{1}{\ln 2}$ is the only critical point. Moreover, note that $f'(x) < 0$ for $x < -\frac{1}{\ln 2}$ and $f'(x) > 0$ for $x > -\frac{1}{\ln 2}$. Hence the absolute minimum value of $f(x)$ occurs at $x = -\frac{1}{\ln 2}$ and is $f(-\frac{1}{\ln(2)}) = (-\frac{1}{\ln(2)})2^{-\frac{1}{\ln(2)}} \approx -0.53$.

Problem 5.

(a) $f(x) = x^2 + e^x + x^e + e^2 \Rightarrow f'(x) = 2x + e^x + ex^{e-1}$.

(b) $f(x) = (\pi - \frac{6}{\sqrt{29}})e^x \Rightarrow f'(x) = (\pi - \frac{6}{\sqrt{29}})e^x$

(c) $f(x) = (3e^3)e^x \Rightarrow f'(x) = (3e^3)e^x = 3e^{x+3}$

Problem 1.
$f(x) = \frac{\frac{3}{2}\ln(x)}{x} \Rightarrow f'(x) = \frac{3}{2}\frac{(\frac{1}{x})x-(1)\ln(x)}{x^2} = \frac{3(1-\ln(x))}{2x^2} \Rightarrow.\ f'(e) = \frac{3(1-\ln(e))}{2e^2} = 0$

Problem 2.
$y = \ln(3x^2) = \ln 3 + \ln x^2 = \ln 3 + 2\ln x \Rightarrow y' = \frac{2}{x}.$

Problem 3.
$y = \frac{\ln x^2}{5e^{3x}} = \frac{2}{5}\frac{\ln(x)}{(e^{3x})} \Rightarrow y' = \frac{2}{5}\frac{(\frac{1}{x})(e^{3x})-(3e^{3x})(\ln x)}{(e^{3x})^2} = \frac{2(\frac{1}{x}-3\ln x)}{5e^{3x}}.$

Problem 4.
$y = \frac{\ln x}{x} \Rightarrow y' = \frac{(\frac{1}{x})x-(1)(\ln x)}{x^2} = \frac{1-\ln x}{x^2}.$

Problem 5.
$y = x\ln\left(\frac{1}{x}\right) = x\ln(x^{-1}) = -x\ln x \Rightarrow y' = -\left((1)\ln x + \left(\frac{1}{x}\right)(x)\right) = -\ln x - 1.$

Problem 6.
$f(x) = x\ln\left(\frac{1}{x}\right) = x\ln(x^{-1}) = -x\ln x \Rightarrow f'(x) = -\left((1)\ln x + \left(\frac{1}{x}\right)(x)\right) = -\ln x - 1.$

Problem 7.
$f(x) = \frac{3\ln(3^6x^7)}{\pi} + \frac{3\ln(3^6)}{\pi} = \frac{3\ln(3^6)+3\ln(x^7)}{\pi} + \frac{3\ln(3^6)}{\pi} = \frac{6\ln(3^6)+21\ln x}{\pi} = \Rightarrow f'(x) = \frac{21}{\pi x}.$

Problem 8.
$f(x) = e^{5x}\ln\left(\frac{\pi}{\sqrt{x}}\right) = e^{5x}(\ln\pi - \ln\sqrt{x}) = e^{5x}\left(\ln\pi - \frac{1}{2}\ln x\right) = (e^{5x})\left(\ln\pi - \frac{1}{2}\ln x\right)$
$\Rightarrow f'(x) = (5e^{5x})\left(\ln\pi - \frac{1}{2}\ln(x)\right) + \left(-\frac{1}{2x}\right)(e^{5x}) = e^{5x}\left(5\left(\ln\pi - \frac{\ln x}{2}\right) - \frac{1}{2x}\right).$

Problem 9.
$f(x) = 3^x(\log x) = 3^x\left(\frac{\ln x}{\ln 10}\right) = \left(\frac{3^x\ln x}{\ln 10}\right) \Rightarrow f'(x) = \frac{((\ln 3)3^x)(\ln x) + (3^x)\left(\frac{1}{x}\right)}{\ln 10} = \frac{3^x(x(\ln 3)(\ln x) + 1)}{x\ln 10}.$

Problem 10.

$f(x) = \frac{\ln(2x^3)}{3e^x} = \frac{\ln 2 + 3\ln x}{3e^x} \Rightarrow f'(x) = \frac{\left(\frac{3}{x}\right)(3e^x) - (\ln 2 + 3\ln x)(3e^x)}{9e^{2x}} = \frac{3 - x\ln 2 - 3x\ln x}{3xe^x}.$

Problem 11.
$f(x) = \frac{x+\ln\left(\frac{1}{x}\right)}{x^2} = \frac{x+\ln(x^{-1})}{x^2} = \frac{x-\ln x}{x^2} \Rightarrow f'(x) = \frac{\left(1-\frac{1}{x}\right)(x^2) - (2x)(x-\ln(x))}{x^4} = \frac{x^2 - x - 2x^2 + 2x\ln(x)}{x^4} =$
$\frac{2\ln(x) - x - 1}{x^3}.$

Problem 12.
$f(x) = x^\pi + \pi^x + \ln\left(\frac{\pi}{x}\right) = x^\pi + \pi^x + \ln(\pi x^{-1}) = x^\pi + \pi^x + \ln\pi - \ln(x) \Rightarrow f'(x) = \pi x^{\pi-1} + (\ln\pi)\pi^x - \frac{1}{x}.$

Problem 13.
$f(x) = x^2\left(\ln x + \ln\left(\sqrt{\frac{61}{2x}}\right)\right) = x^2\left(\ln x + \left(\frac{1}{2}\right)\left(\ln\left(\frac{61}{2x}\right)\right)\right) = x^2\left(\ln x + \left(\frac{1}{2}\right)(\ln 61 - \ln(2x))\right) =$
$x^2\left(\ln x + \left(\frac{1}{2}\right)(\ln 61 - \ln 2 - \ln x)\right) = \left(\frac{1}{2}\right)x^2(\ln 61 - \ln 2 + \ln x) \Rightarrow$
$f'(x) = \left(\frac{1}{2}\right)\left[(2x)(\ln 61 - \ln 2 + \ln x) + x^2\left(\frac{1}{x}\right)\right] = x(\ln 61 - \ln 2 + \ln x) + \frac{x}{2}.$

Problem 16.

(a) $y = 3\ln 5x + 6\ln\left(\frac{3}{x}\right) = 3\ln 5 + 3\ln x + 6\ln 3 - 6\ln x = 3\ln 5 + 6\ln 3 - 3\ln x \Rightarrow y' = -\frac{3}{x}.$

(b) $y = 20\log\left(\frac{x}{100}\right) = 20\log x - 20\log(100) \Rightarrow y' = \frac{20}{x\ln 10}$

(c) $y = \frac{ke^{2kx}}{\sqrt{k+1}} = \frac{k}{\sqrt{k+1}}e^{2kx} \Rightarrow y' = \frac{k}{\sqrt{k+1}}(2k)e^{2kx} = \frac{2k^2}{\sqrt{k+1}}e^{2kx}.$

(d) $y = \frac{(\ln 2)e^{5x}}{\ln 4} + \frac{(\ln 2)e^{\ln 2}}{\ln 3} = \frac{(\ln 2)e^{5x}}{\ln 4} + \frac{2\ln 2}{\ln 3} \Rightarrow y' = \frac{(5\ln 2)e^{5x}}{\ln 4}.$

Problem 19.

(a) $P(t) = P_0 e^{kt}$, for some constant k. Now $P(0) = 60 = P_0 e^{k(0)} = P_0$, and hence $P(30) = 2000 = 60e^{30k}$. Thus $\frac{100}{3} = e^{30k} \Rightarrow 30k = \ln\left(\frac{100}{3}\right) \Rightarrow k = \frac{1}{30}\ln\left(\frac{100}{3}\right) \Rightarrow k \approx 0.1169$. Therefore, $P(t) = 60e^{0.1169t}$.

(b) $\frac{P(t+1)}{P(t)} = \frac{e^{0.1169(t+1)}}{e^{0.1169t}} = e^{0.1169} \approx 1.1169$. Hence the annual growth rate was about 11.69%.

(c) The doubling time is the value of t for which $2P_0 = P(t) = P_0 e^{0.1169t}$. (Note here that this last equation implies that the doubling time is independent of the particular value of P_0.) Now $2P_0 = P_0 e^{0.1169t} \Rightarrow 2 = e^{0.1169t} \Rightarrow \ln 2 = 0.1169t \Rightarrow t = \frac{\ln 2}{0.1169} \approx 5.93$. The doubling time is 5.93 years.

(d) $P'(t) = 60(0.1169)e^{0.1169t} = 7.014e^{0.1169t} \Rightarrow P'(0) = 7.014e^{0.1169(0)} = 7.014$ oryx per year and $P'(30) = 7.014e^{0.1169(30)} \approx 233.903$ oryx per year.

(e) The average rate of change over the 30-year period is $\frac{P(30)-P(0)}{30-0} = \frac{2000-60}{30} \approx 64.667$ oryx/year.

(f) (i) The oryx population ten years after their introduction

(ii) The number of years after the oryx introduction it took the population to grow to 200 oryx

(g) $P^{-1}(200)$ is the value of t for which $200 = P(t) = 70e^{0.1169t}$. From this equation, we have $\frac{20}{7} = e^{0.1169t} \Rightarrow t = \frac{1}{0.1169}\ln\left(\frac{20}{7}\right) \approx 8.98$. Thus $P^{-1}(200) \approx 8.98$.

Problem 21.

$P(2) = 1000 = P_0 e^{2k}$ and $P(4) = 1300 = P_0 e^{4k}$. Dividing the second equation by the first, we obtain $\frac{1300}{1000} = \frac{P_0 e^{4k}}{P_0 e^{2k}} \Rightarrow 1.3 = e^{2k} \Rightarrow 2k = \ln 1.3 \Rightarrow k = 0.5 \ln 1.3 \approx 0.131$. Substituting this value of k into the first equation, we have $1000 = P_0 e^{2(0.131)} \Rightarrow P_0 \approx 769.5$. Therefore, the growth equation is $P(t) = 769.5e^{0.131t}$.

16.1

Problem 5.

$f(x) = e^x(3x^2+1)^{-1} \Rightarrow f'(x) = e^x((3x^2+1)^{-1}) + \left(-(3x^2+1)^{-2}\left(\frac{d}{dx}(3x^2+1)\right)\right)e^x = e^x((3x^2+1)^{-1} - 6x(3x^2+1)^{-2}) = \frac{e^x(3x^2-6x+1)}{(3x^2+1)^2}$.

Problem 8.

$f(x) = \ln(\sqrt{x^3})e^{6x} = \frac{3}{2}(\ln x)e^{6x} \Rightarrow f'(x) = \frac{3}{2x}e^{6x} + \frac{3}{2}(\ln x)(6e^{6x}) = \frac{3e^{6x}(1+6x\ln x)}{2x}$.

Problem 11.

$f(x) = \frac{e^{\pi x}}{(x+x^2)^3} = e^{\pi x}(x+x^2)^{-3} \Rightarrow f'(x) = \left(e^{\pi x}\left(\frac{d}{dx}\pi x\right)\right)(x+x^2)^{-3} + \left(-3(x+x^2)^{-4}\left(\frac{d}{dx}(x+x^2)\right)\right)e^{\pi x} = \pi e^{\pi x}(x+x^2)^{-3} - 3(x+x^2)^{-4}(1+2x)e^{\pi x} = e^{\pi x}(\pi(x+x^2)^{-3} - 3(x+x^2)^{-4}(1+2x))$.

Problem 13.

$f(x) = \frac{1}{x^3+7x+5} = (x^3+7x+5)^{-1} \Rightarrow f'(x) = -(x^3+7x+5)^{-2}\left(\frac{d}{dx}(x^3+7x+5)\right) = -(x^3+7x+5)^{-2}(3x^2+7)$.

Problem 17.

$f(x) = \sqrt{e^x + \ln(x+1)^2} = (e^x + 2\ln(x+1))^{1/2} \Rightarrow f'(x) = \frac{1}{2}(e^x+2\ln(x+1))^{-1/2}\left(\frac{d}{dx}(e^x + 2\ln(x+1))\right) = \frac{1}{2}(e^x + 2\ln(x+1))^{-\frac{1}{2}}\left(e^x + \frac{2}{x+1}\left(\frac{d}{dx}(x+1)\right)\right) = \frac{1}{2}(e^x+2\ln(x+1))^{-\frac{1}{2}}\left(e^x + \frac{2}{x+1}\right) = \frac{xe^x + e^x + 2}{2x(x+1)\sqrt{e^x+\ln(x+1)^2}}$.

Problem 18.

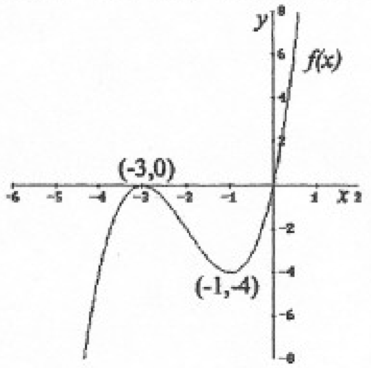
$f(x) = \frac{1}{\ln(x^2+2)} = (\ln(x^2+2))^{-1} \Rightarrow f'(x) = -(\ln(x^2+2))^{-2}\left(\frac{d}{dx}(\ln(x^2+2))\right) = -(\ln(x^2+2))^{-2}\left(\frac{1}{x^2+2}\left(\frac{d}{dx}(x^2+2)\right)\right) = -(\ln(x^2+2))^{-2}\left(\frac{2x}{x^2+2}\right) = -\frac{2x}{(x^2+2)(\ln(x^2+2))^2}$.

Problem 22.

$f(x) = x(x+3)^2 = x^3 + 6x^2 + 9x \Rightarrow$

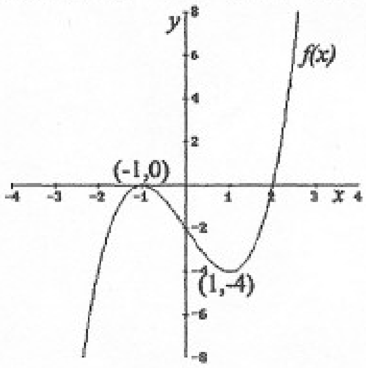
$$f'(x) = (1)(x+3)^2 + x(2)(x+3) = (x+3)((x+3)+2x) = 3(x+3)(x+1) = 3x^2 + 12x + 9.$$

$f'(x) = 0$ when $x = -3$ or $x = -1$. Now $f''(x) = 6x + 12$, and hence $f''(-3) = 6(-3) + 12 = -6 < 0$ and $f''(-1) = 6(-1) + 12 = 18 > 0$. Therefore, $(-3, f(-3)) = (-3, 0)$ is a local maximum point and $(-1, f(-1)) = (1, -4)$ is a local minimum point.



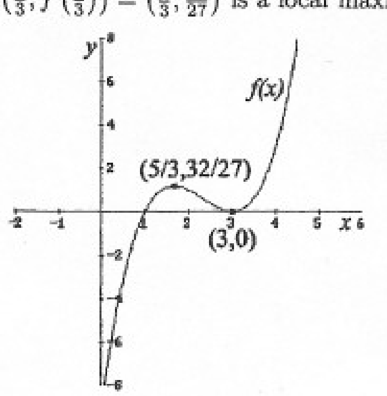
Problem 23.

$f(x) = (x-2)(x+1)^2 = (x-2)(x^2+2x+1) \Rightarrow f'(x) = (1)(x+1)^2 + (x-2)(2)(x+1) = (x+1)((x+1) + 2(x-2)) = 3(x+1)(x-1) = 3x^2 - 3$. $f'(x) = 0$ when $x = -1$ or $x = 1$. Now $f''(x) = 6x$, and hence $f''(-1) = -6 < 0$ and $f''(1) = 6 > 0$. Therefore $(-1, f(-1)) = (-1, 0)$ is a local maximum point and $(1, f(1)) = (1, -4)$ is a local minimum point.



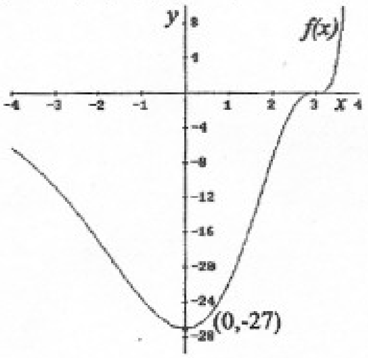
Problem 24.

$f(x) = (3-x)^2(x-1) = (x-3)^2(x-1) \Rightarrow f'(x) = 2(x-3)(x-1) + (x-3)^2(1) = (x-3)(2(x-1)+(x-3)) = (x-3)(3x-5) = 3x^2 - 14x + 15$. $f'(x) = 0$ when $x = 3$ or $x = \frac{5}{3}$. Now $f''(x) = 6x - 14$, and hence $f''(3) = 4 > 0$ and $f''\left(\frac{5}{3}\right) = -4 < 0$. Therefore $(3, f(3)) = (3, 0)$ is a local minimum point and $\left(\frac{5}{3}, f\left(\frac{5}{3}\right)\right) = \left(\frac{5}{3}, \frac{32}{27}\right)$ is a local maximum point.



Problem 25.

$f(x) = e^x(x-3)^3 \Rightarrow f'(x) = e^x(x-3)^3 + 3(x-3)^2(e^x) = x(x-3)^2e^x$. $f'(x) = 0$ when $x = 0$ or $x = 3$. Note that $f'(x) > 0$ on $(0,3) \cup (3,\infty)$ and hence there is not local extremum at $x = 3$. As $f'(x) < 0$ on $(-\infty, 0)$, $(0, f(0)) = (0, -27)$ is a local minimum point.



16.3

Problem 5.

(a) $y' = e^x x^e + e x^{e-1} e^x = e^x(x^e + e x^{e-1}) = e^x x^e + e^{x+1} x^{e-1}$.

(b) $y' = e^{1/x}(-x^{-2}) = -\frac{e^{1/x}}{x^2}$.

(c) $y = \sqrt{e^{-x}x} = e^{-x/2}x^{1/2} \Rightarrow$
$y' = -\frac{1}{2}e^{-x/2}(x^{1/2}) + e^{-x/2}\left(\frac{1}{2}\right)x^{-1/2} = \frac{1-x}{\sqrt{2e^{-x}x}}$.

(d) $y = \left[\ln\sqrt{1-x}\right]^{-3.5} = \left[\frac{1}{2}\ln(1-x)\right]^{-3.5} \Rightarrow$
$y' = (-3.5)\left[\frac{1}{2}\ln(1-x)\right]^{-4.5}\left(-\frac{1}{2(1-x)}\right) = \frac{7}{4(1-x)\left[\ln\sqrt{1-x}\right]^{4.5}}$.

(e) $y = \ln\left(\frac{x+1}{x-1}\right) = \ln(x+1) - \ln(x-1) \Rightarrow y' = \frac{1}{x+1}(1) - \frac{1}{x-1}(1) = \frac{1}{x+1} - \frac{1}{x-1} = -\frac{2}{x^2-1}$.

(f) $y' = \frac{5}{4}(1-\ln(x))^{1/4}\left(-\frac{1}{x}\right) = -\frac{5(1-\ln(x))^{1/4}}{4x}$.

(g) $y = \ln\sqrt{x(x+1)} = \frac{1}{2}(\ln(x) + \ln(x+1)) \Rightarrow y' = \frac{1}{2}\left(\frac{1}{x} + \frac{1}{x+1}(1)\right) = \frac{2x+1}{2x^2+2x}$.

(h) $y = \frac{5}{\sqrt{e^{1/(6x)+x}}} = 5e^{(-1/2)(1/(6x)+x)} = 5e^{-1/(12x)-x/2} \Rightarrow$
$y' = 5\left(\frac{1}{12x^2} - \frac{1}{2}\right)e^{-1/(12x)-x/2} = \left(\frac{5-30x^2}{12x^2}\right)e^{-1/(12x)-x/2}$.

16.3

Problem 4.

(a) $y = \frac{1}{x\ln 2+1} = (x\ln 2+1)^{-1} \Rightarrow y' = -(\ln 2)(x\ln 2+1)^{-2} = -\frac{\ln 2}{(x\ln 2+1)^2}$.

(b) $y' = \left(\frac{1}{5x^3+8x}\right)(15x^2+8) = \frac{15x^2+8}{5x^3+8x}$.

(c) $y' = (\ln 2)2^x(x^2+x)^7 + 7(x^2+x)^6(2x+1))(2^x) =$
$2^x(x^2+x)^6(x^2\ln 2 + x\ln 2 + 14x + 7)$.

(d) $y = \sqrt{\ln 5x + e^{6x}} = \sqrt{\ln 5 + \ln x + e^{6x}} \Rightarrow y' = \frac{1}{2}(\ln(5) + \ln(x) + e^{6x})^{-\frac{1}{2}}\left(\frac{1}{x} + 6e^{6x}\right)$.

(e) $y = \frac{7}{\sqrt{\ln x}} = 7(\ln x)^{-1/2} \Rightarrow y' = 7\left(-\frac{1}{2}(\ln x)^{-3/2}\left(\frac{1}{x}\right)\right) = -\frac{7}{2x(\ln x)^{3/2}}$.

(f) $y' = 4^{x/3}(\ln 4)(\ln 3x)\left(\frac{1}{3}\right) + 4^{x/3}\frac{3}{3x} = 4^{x/3}\left(\frac{1}{3}\ln 4\ln 3x + \frac{1}{x}\right)$.

Problem 5

(a) $y = (2^2)^x = 4^x \Rightarrow y' = (\ln 4)4^x$.

(b) $y' = (\ln 2)2^{2^x}(\ln 2)2^x = (\ln(2))^2 2^{2^x+x}$.

(c) $y = \frac{e^{\pi x}}{x} = x^{-1}e^{\pi x} \Rightarrow y' = -x^{-2}e^{\pi x} + \pi e^{\pi x}(x^{-1}) = e^{\pi x}\left(\frac{\pi}{x} - \frac{1}{x^2}\right)$.

(d) $y = \frac{x^3+1}{x^2+1} = (x^3+1)(x^2+1)^{-1} \Rightarrow$
$y' = 3x^2((x^2+1)^{-1}) - (x^2+1)^{-2}(2x)(x^3+1) = \frac{3x^2}{x^2+1} - \frac{2x(x^3+1)}{(x^2+1)^2}$.

(e) $y = 5\ln\left(\frac{5x+3}{\sqrt{x}}\right) = 5\ln(5x^{1/2} + 3x^{-1/2}) \Rightarrow y' = 5\left(\frac{1}{5x^{1/2}+3x^{-1/2}}\right)\left(\frac{5}{2}x^{-1/2} - \frac{3}{2}x^{-3/2}\right) = \frac{5(5x+3)}{2(5x^2+3x)} = \frac{25x+15}{10x^2+6x}$.

(f) $y = \frac{3}{2\ln(8x^2+1)} = \frac{3}{2}(\ln(8x^2+1))^{-1} \Rightarrow y' = \frac{3}{2}\left(-\frac{1}{8x^2+1}\right)(\ln(8x^2+1))^{-2})(16x) = -\frac{24x}{(8x^2+1)\ln(8x^2+1))^2}$.

Problem 6.
The denominator is always at least 1 since $x^2 + 1 \geq 1$ for all x, so the function is no larger than 3. It attains this value at $x = 0$.
Now $f(x) = \frac{3}{\sqrt{x^2+1}} = 3(x^2+1)^{-1/2} \Rightarrow f'(x) = -\frac{3}{2}(x^2+1)^{-3/2}(2x) = -3x(x^2+1)^{-3/2}$. Now $f'(x) = 0$ when $x = 0$, and $f'(x) > 0$ for $x < 0$ and $f'(x) < 0$ for $x > 0$. Hence the global maximum value, 3, of f is attained at $x = 0$.

Problem 13.
$h'(x) = f'(\ln x)\left(\frac{1}{x}\right) - \frac{1}{f(x)}(f'(x)) = \frac{f'(\ln x)}{x} - \frac{f'(x)}{f(x)}$.

Problem 14.
$h'(x) = \frac{1}{2\sqrt{f(x)g(x)}}(f'(x)g(x) + f(x)g'(x)) = \frac{f'(x)g(x)+f(x)g'(x)}{2\sqrt{f(x)g(x)}}$.

Problem 15.
$h'(x) = -\frac{1}{2}(f(g(x)))^{-3/2}(f'(g(x))g'(x)$.

Problem 16.
$h'(x) = f'(x^2)2xe^{g(x)} + f(x^2)g'(x)e^{g(x)} = e^{g(x)}(2xf'(x^2) + f(x^2)g'(x))$.

Problem 17.
$h'(x) = -\frac{2f'(x)}{[f(x)]^3} - f'\left(\frac{1}{x^2}\right)\left(\frac{2}{x^3}\right)$.

Problem 18.
$h'(x) = 3[f(x)]^2f'(x)g(2x) + 2[f(x)]^3g'(2x)$.

Problem 23.
$y' = \left(\frac{f(x^2)}{x}\right)\left(\frac{(1)f(x^2)-xf'(x^2)(2x)}{[f(x^2)]^2}\right) = \frac{f(x^2)-2x^2f'(x^2)}{xf(x^2)}$.

Problem 24.
$y' = 2f(x)(f'(x)) + (\ln 2)2^{f(x)}f'(x) = f'(x)(2f(x) + (\ln 2)2^{f(x)})$.

Problem 25.
$y = \ln\left(\frac{x\cdot f(x)}{\sqrt{3x^3+2}}\right) = \ln(x\cdot f(x)) - \ln(\sqrt{3x^3+2}) = \ln x + \ln f(x) - \frac{1}{2}\ln(3x^3+2) \Rightarrow$
$y' = \frac{1}{x} + \frac{f'(x)}{f(x)} - \frac{1}{2}\left(\frac{9x^2}{3x^3+2}\right) = \frac{1}{x} + \frac{f'(x)}{f(x)} - \frac{9x^2}{6x^3+4}$.

Problem 26.
$f(x) = \ln(e^{(x+5)^2}) = (x+5)^2 \Rightarrow f'(x) = 2(x+5)(1) = 2x + 10$.

Problem 27.
$f'(x) = e^{(g(x))^2}(2g(x)g'(x)) = 2g(x)g'(x)e^{(g(x))^2}$.

Problem 28.
$f'(x) = \pi(x^3+e)^{\pi-1}(3x^2) = 3\pi x^2(x^3+e)^{\pi-1}$.

Problem 29.

(a) $\frac{f(2.0001)-f(2)}{0.0001} \approx 6.773$

(b) $2(2^{2-1}) = 4 \neq 6.773$; $f'(x) \neq x \cdot x^{x-1}$ because the exponent x is not a constant.

(c) $\ln 2 \cdot 2^2 \approx 2.773 \neq 6.773$; $f'(x) \neq \ln x \cdot x^x$ because the base x is not a constant.

(d) $f(x) = x^x = e^{\ln x^x} = e^{x\ln x} \Rightarrow f'(x) = e^{x\ln x}((1)(\ln x) + \left(\frac{1}{x}\right)(x)) = e^{x\ln x}(\ln x + 1)$.

Problem 35.

(a) $A(t) = \pi(r(t))^2$. Therefore, $A'(t) = \pi(2r(t))r'(t) = 2\pi r(t)r'(t)$.

(b) $V(t) = \pi(r(t))^2h(t)$. Therefore, $V'(t) = \pi[2\pi r(t)r'(t)h(t) + (r(t))^2h'(t)]$.