Name: SOLUTIONS

Math Xa Midterm Examination I—Thursday, October 28, 2004

Please circle your section:

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MWF 11–12	MWF 10–11	MWF 10–11	MWF 11–12

Problem Number	Possible Points	Score
1	6	
2	15	
3	10	
4	5	
5	8	
6	20	
7	8	
8	10	
9	8	
10	10	
Total	100	

Directions—Please Read Carefully! You have two hours to take this midterm. Make sure to use correct mathematical notation. Any answer in decimal form must be accurate to three decimal places, unless otherwise specified. Pace yourself by keeping track of how many problems you have left to go and how much time remains. You do not have to answer the problems in any particular order, so move to another problem if you find you are stuck or spending too much time on a single problem. To receive full credit on a problem, you will need to justify your answers carefully—unsubstantiated answers will receive little or no credit (except if the directions for that question specifically say no justification is necessary, such as in a True/False section). Please be sure to write neatly—illegible answers will receive little or no credit. If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of this on the problem page so that the grader knows where to find your answers. You may use a calculator on this exam, but no other aids are allowed. **Good Luck!!!**

1. (6 points) Suppose that you are working with laboratory rats as part of a biology experiment. Let $f(t)$ be a function which gives the temperature, in degrees Celsius, of a laboratory rat with respect to time, in hours. If the experiment began at time $t = 0$ and ended at time $t = c$, express each of the following in terms of functional notation.

   (a) The temperature of the rat at the end of the experiment.

   $f(c)$

   (b) The temperature of the rat an hour before the experiment ended.

   $f(c-1)$

   (c) Ten degrees more than the temperature of the rat at the end of the experiment.

   $f(c) + 10$

   (d) Half of the temperature of the rat at the end of the experiment.

   $\frac{f(c)}{2}$

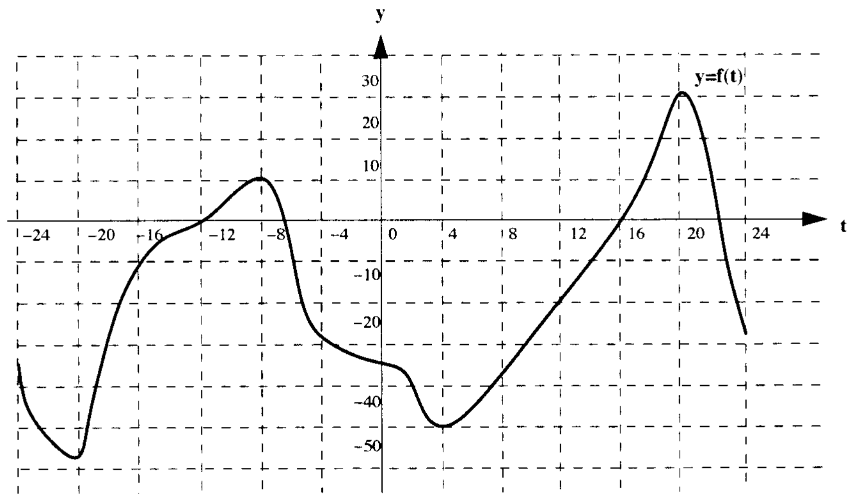
   (e) The temperature of the rat half-way through the experiment.

   $f\left(\frac{c}{2}\right)$

   (f) The average temperature of the rat over the entire experiment.

   DISCARDED QUESTION

2. (15 points) The graph of a function $g(t)$ below represents the official temperature in a small town in Montana over a period of 48 hours last winter. The time $t = 0$ represents midnight on January 16 (so the times $t > 0$ correspond to times on January 16 and the times $t < 0$ correspond to times on January 15).



(a) What is the temperature at noon on the 16th?

− 15 degrees

(b) For what values of t is the function increasing?

$[-20, -8]$ and $[4, 20]$

(c) Find the average rate of change of the temperature from noon January 15th to 4 a.m. on January 16th.

$$\frac{\Delta g}{\Delta t} = \frac{-50-0}{4-(-12)} = \frac{-50}{16} = -3.125$$

(d) A retailer in nearby North Dakota decides to base the price it charges for long underwear on the temperature. The retailer uses the function $p(x)$, where x is the temperature in degrees Celsius and p is the price of long underwear in dollars. Values of p for various x are given in the table below.

Temperature x	Price p
30° C	\$ 10
20° C	\$ 15
10° C	\$ 20
0° C	\$ 30
−10° C	\$ 40
−20° C	\$ 50
−30° C	\$ 60
−40° C	\$ 80
−50° C	\$ 100

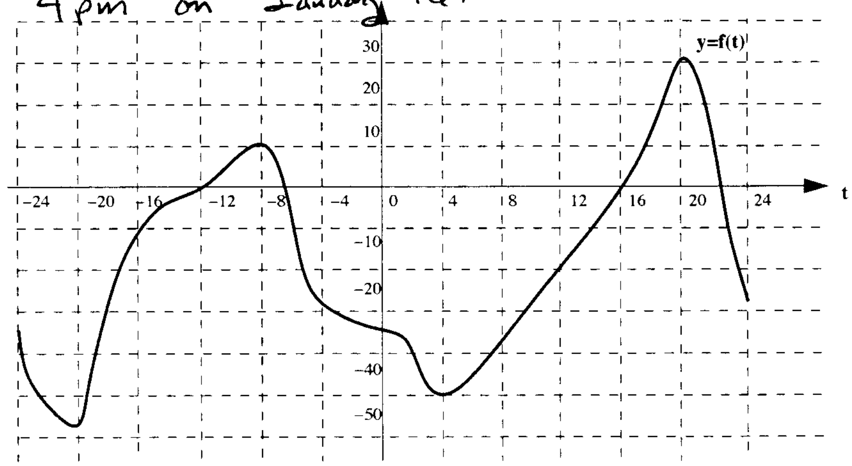
At 8 a. m. on January 15th, what price did the retailer charge for long underwear?

$$p(g(-16)) = p(-10) = \$40$$

(e) Compute $p(g(16))$. Explain what you have computed without using any mathematical terminology.

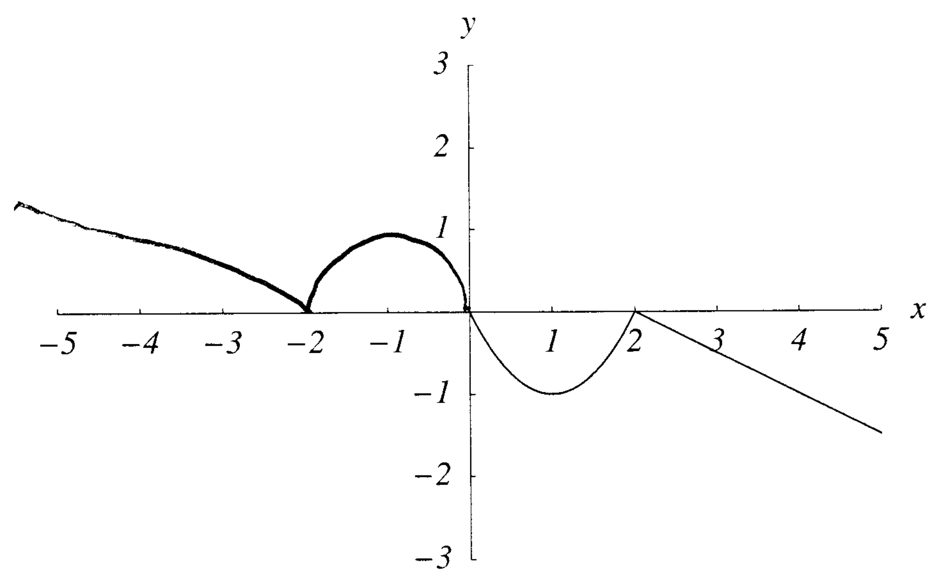
$$p(g(16)) = p(0) = 30$$

We have computed the price of long underwear at 4pm on January 16.

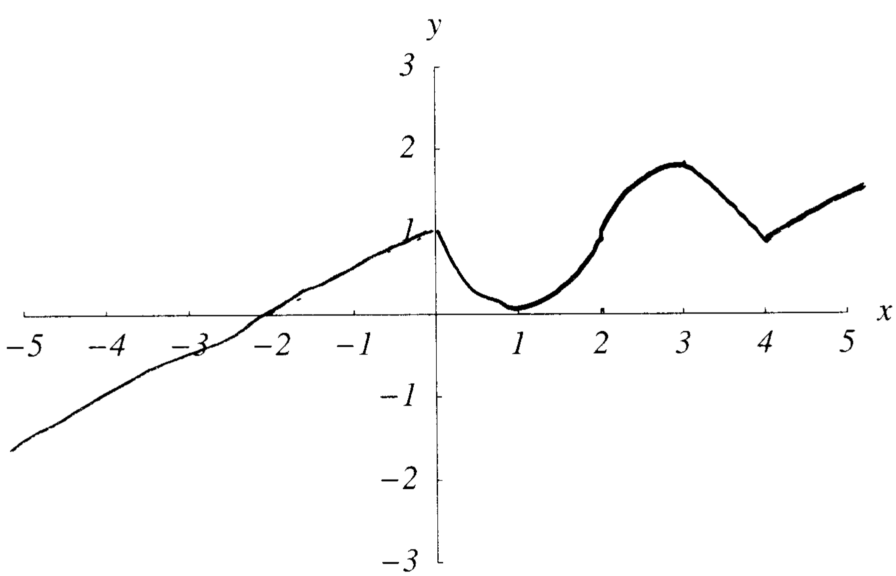


3. (10 points) Recall that an *odd function* is one that is symmetric about the origin. Equivalently, we can say that f is an odd function if it satisfies the relation $f(-x) = -f(x)$.

(a) Suppose that g is an odd function. The function g is defined for all numbers x, but in the graph below it is shown only for $x \geq 0$. Complete the graph of g.



(b) Now sketch the graph of $y = -g(x-2) + 1$ on the axes below.



4. (5 points) If $f(x) = x^3 + x - 1$, find

$$\frac{f(2+h) - f(2)}{h}.$$

Be sure to show each step of your calculation and simplify your answer.

$$\frac{f(2+h) - f(2)}{h}$$

$$= \frac{\left[(2+h)^3 + (2+h) - 1\right] - \left[2^3 + 2 - 1\right]}{h}$$

$$= \frac{\cancel{2^3} + 3\cdot 2^2 h + 3\cdot 2h^2 + h^3 + \cancel{2} + h - \cancel{1} - \cancel{2^3} - \cancel{2} \cancel{+1}}{h}$$

$$= \frac{12h + 6h^2 + h^3 + h}{h} = 13 + 6h + h^2$$

5. (8 points) Bill Gates was born on October 28th, 1955. (Happy 49th Birthday Bill!) Since that time, Bill has accumulated a considerable fortune. Let $f(n)$ be the amount of money that Bill made between his n-th and $(n+1)$-th birthday, measured in millions of dollars. For example, $f(0)$ is the amount of money that Bill made from the time he was born up until his first birthday, and $f(48)$ is the amount of money that Bill made during the last year.

   (a) Express the amount of money in millions of dollars that Bill made during the year following his drop-out from Harvard in 1976 in terms of the function f. (For simplicity assume that he dropped out on his birthday.)

   $$f(21)$$

   (b) Express the average amount of money in millions of dollars that Bill made per year between his 10th and 15th birthdays in terms of the function f.

   $$\frac{f(10) + f(11) + f(12) + f(13) + f(14)}{5}$$

   (c) Now let $g(n)$ be the function that gives the total amount of money that Bill had on his nth birthday, measured in millions of dollars. For example, the equation $g(49) = 63,307$ means that today Bill has a bit more than 63 billion dollars. Express the amount of money in millions of dollars that Bill made during the year following his drop-out from Harvard in 1976 in terms of the function g. (For simplicity assume that he dropped out on his birthday.)

   $$g(22) - g(21)$$

   (d) Express the average amount of money in millions of dollars that Bill made per year between his 10th and 15th birthdays in terms of the function g.

   $$\frac{g(15) - g(10)}{5}$$

6. (20 points) The famous author John Uptight must decide between two publishers who are vying for the rights to his new book, *Zen and the Art of Taxidermy.* Publisher A offers royalties of 1% of the net proceeds on the first 30,000 copies sold and 3.5% on all copies in excess of 30,000 copies. Publisher A expects to net \$2 on each copy sold. Publisher B will pay no royalties on the first 4,000 copies sold but will pay 2% on the net proceeds of all copies sold in excess of 4,000 copies. Publisher B expects to net \$3 on each copy sold.

(a) Express the revenue that the author should expect if he signs with Publisher A as a function P_A of the number of books sold, x.

$$P_A(x) = \begin{cases} 0.01(2)x, & 0 \le x \le 30{,}000 \\ 600 + 0.035(2)(x - 30{,}000), & x > 30{,}000 \end{cases}$$

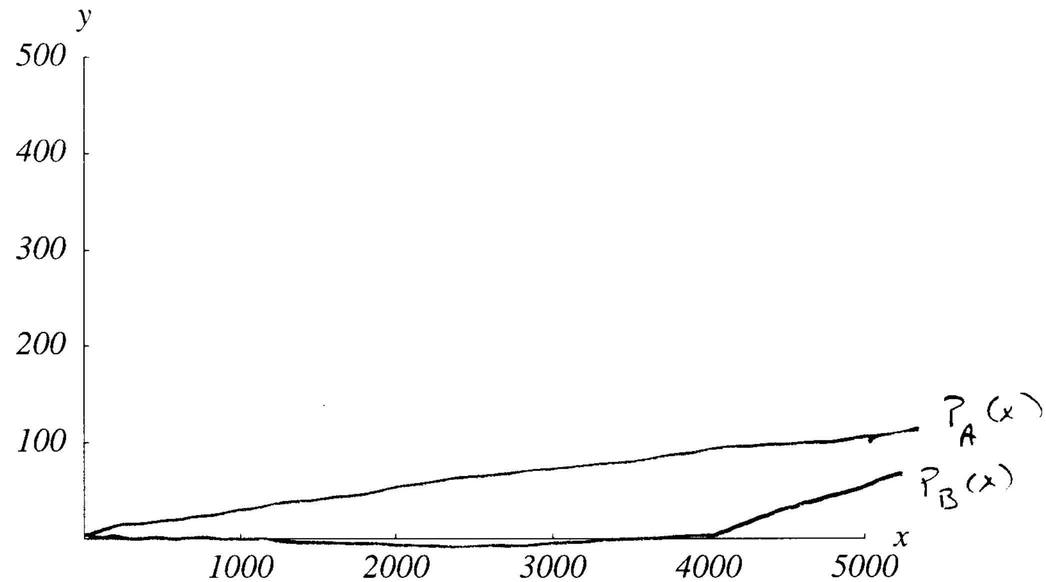
$$= \begin{cases} 0.02x, & 0 \le x \le 30{,}000 \\ 0.07x - 1500, & x > 30{,}000 \end{cases}$$

(b) Express the revenue that the author should expect if he signs with Publisher B as a function P_B of the number of books sold, x.

$$P_B(x) = \begin{cases} 0, & 0 \le x \le 4000 \\ 0.02(3)(x - 4000), & x > 4000 \end{cases}$$

$$= \begin{cases} 0, & 0 \le x \le 4000 \\ 0.06x - 240, & x > 4000 \end{cases}$$

(c) Sketch the graphs of both P_A and P_B for $0 \leq x \leq 5000$ on the coordinate axis below.



(d) For what values of x are the two offers equivalent?

$X = 6000$

$X = 126{,}000$ copies

GO WITH PUBLISHER B IF $6{,}000 \leq X \leq 126{,}000$

7. (8 points)

(a) Find the equation of a line that is *parallel* to $2y - 4x = 3$ passing through the point $(a, -2)$.

$$2y - 4x = 3 \implies 2y = 4x + 3 \implies y = 2x + \frac{3}{2}$$

$$\implies y - (-2) = 2(x - a)$$

$$\implies y + 2 = 2x - 2a$$

$$\implies \boxed{y = 2x - 2a - 2}$$

(b) Find the equation of a line that is *perpendicular* to $2y - 4x = 3$ passing through the point $(a, -2)$.

$$y - (-2) = -\frac{1}{2}(x - a)$$

$$y + 2 = -\frac{1}{2}x + \frac{1}{2}a$$

$$\boxed{y = -\frac{1}{2}x + \frac{1}{2}a - 2}$$

8. (10 points) Which of the following rules represents a function? For each rule that is not a function, explain why not.

   (a) The rule that takes the speed of a car driving along Massachusetts Avenue and returns the position of the car.

   NOT A FUNCTION – A CAR CAN DRIVE AT THE SAME SPEED AT TWO DIFFERENT POSITIONS ON MASSACHUSETTS AVENUE.

   (b) The rule that takes the position of a car driving along Massachusetts Avenue and returns the speed of the car.

   FUNCTION

   (c) The rule that takes as input a number of pages, and returns the title of a book with that many pages.

   NOT A FUNCTION. THERE CAN BE MANY BOOKS WITH THE SAME NUMBER OF PAGES

   (d) The rule that takes as input the specific edition of a book and returns the number of pages in the book.

   FUNCTION

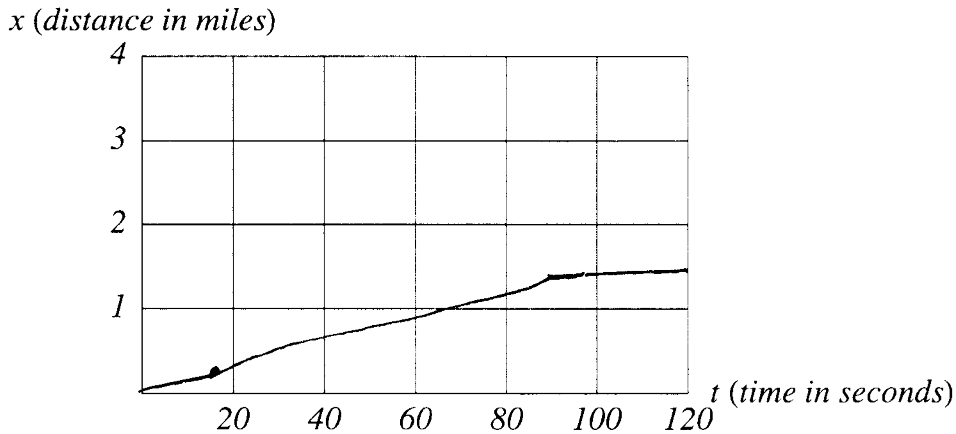
   (e) The rule that takes the number of times a grandfather clock chimes as input and returns the *specific* time of day as its output.

   NOT A FUNCTION. THE CLOCK WILL SOUND THE SAME OF CHIMES AT 1 AM AND 1 PM.

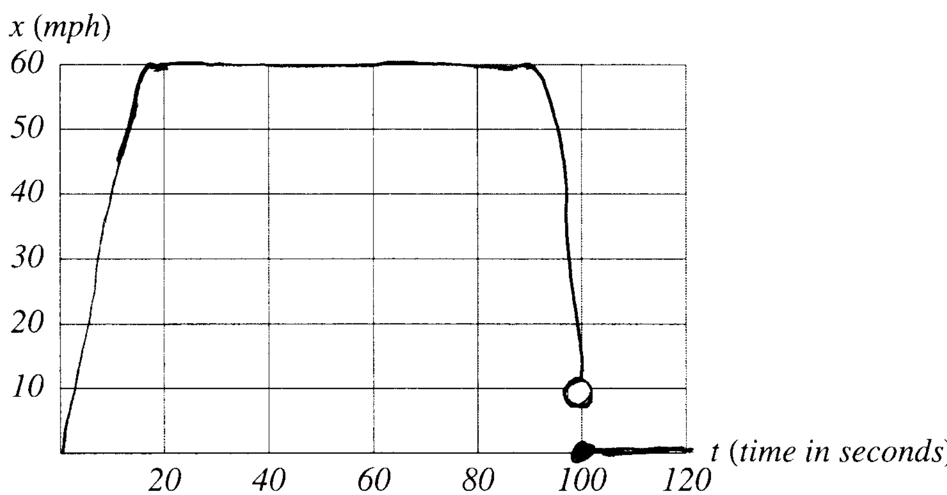
9. (8 points) After a braking test on a straight track on the Utah Salt Flats, the driver reported:

> I accelerated smoothly from rest. After 15 seconds, I was cruising at 60 mph. After 90 seconds, I hit the brakes. The car decelerated, but I couldn't stop in time. I hit the crash barrier 10 seconds later at 10 mph.

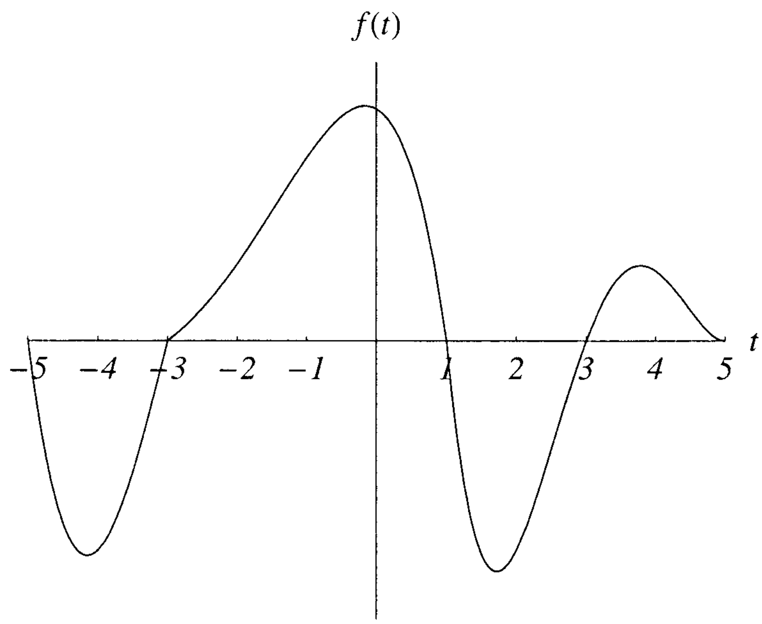
(a) Draw a plausible graph of the distance traveled by the car as a function of time.



(b) Draw a plausible graph of the car's speed as a function of time.



10. (10 points) Interstate 80 runs in an east-west direction through southern Wyoming. Trooper Smith, a state patrolman based in Rawlins on I-80, regularly patrols the Interstate. Consider the graph of $f(t)$, given below, which describes the motion of Trooper Smith's patrol car along the Interstate between 7:00 a.m. and 5:00 p.m. on October 28, 2004, where $t = 0$ corresponds to noon, $t = -5$ corresponds to 7:00 a.m., and $t = 5$ corresponds to 5:00 p.m.



(a) Suppose that $f(t)$ gives Trooper Smith's position in miles from Rawlins, where $f = 0$ when she is at Rawlins, $f < 0$ when she is west of Rawlins, and $f > 0$ when she is east of Rawlins. When is the trooper driving towards Rawlins?

$(-4, -3)$, $(0, 1)$, $(2, 3)$, $(3.75, 5)$

(b) When does the trooper change direction if f is the *position* function?

AT -4, -0.25, 2, 3.75

(c) Now suppose that $f(t)$ gives Trooper Smith's velocity in miles per hour, where the trooper's car is headed west if $f < 0$ and headed east if $f > 0$. When is the trooper driving towards Rawlins?

DISCARDED QUESTION.

(d) When does the trooper change direction if f is the *velocity* function?

AT -3, 1, 3

(e) What direction is the trooper going at noon if f is the *velocity* function?

EAST