Math Xa
Review Session #2

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1/12/05

Second Third of the Class

- The derivative as the slope of a tangent line to the graph of a function.
- The derivative as a rate of change.
- The formal definition of a derivative.
- The meaning of the second derivative.

Second Third of the Class (continued)

- Limits and continuity.
- Sketching the graph of a curve using the derivative.
- The rules of differentiation including the chain rule.
- Linear approximation.

Second Third of the Class (continued)

- The exponential function and its derivative.
- First and second derivative tests.
- Optimization.
- Polynomial and rational functions.

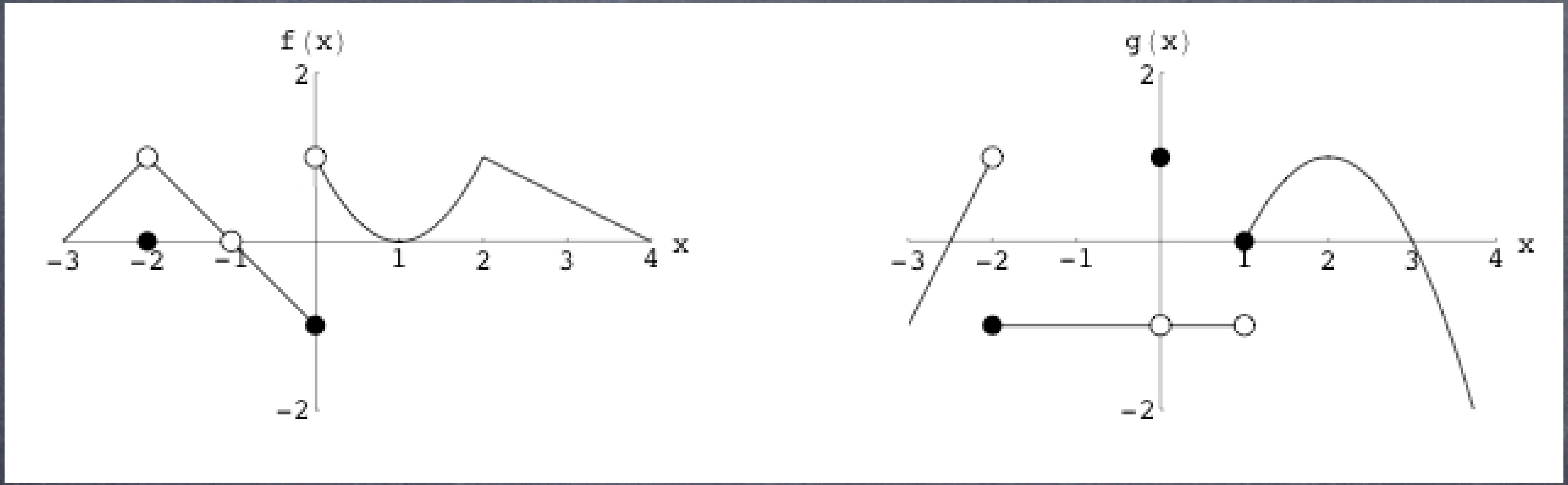
The rules of differentiation

- $\frac{d}{dx}k = 0$, where k is constant.
- $\frac{d}{dx}kf(x) = k\frac{d}{dx}f(x)$, where k is constant.
- $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$.
- $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$.
- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$.
- $\frac{d}{dx}x^n = nx^{n-1}$, where n is an integer.
- $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$.
- $\frac{d}{dx}e^{kx} = ke^{kx}$.
- $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$.

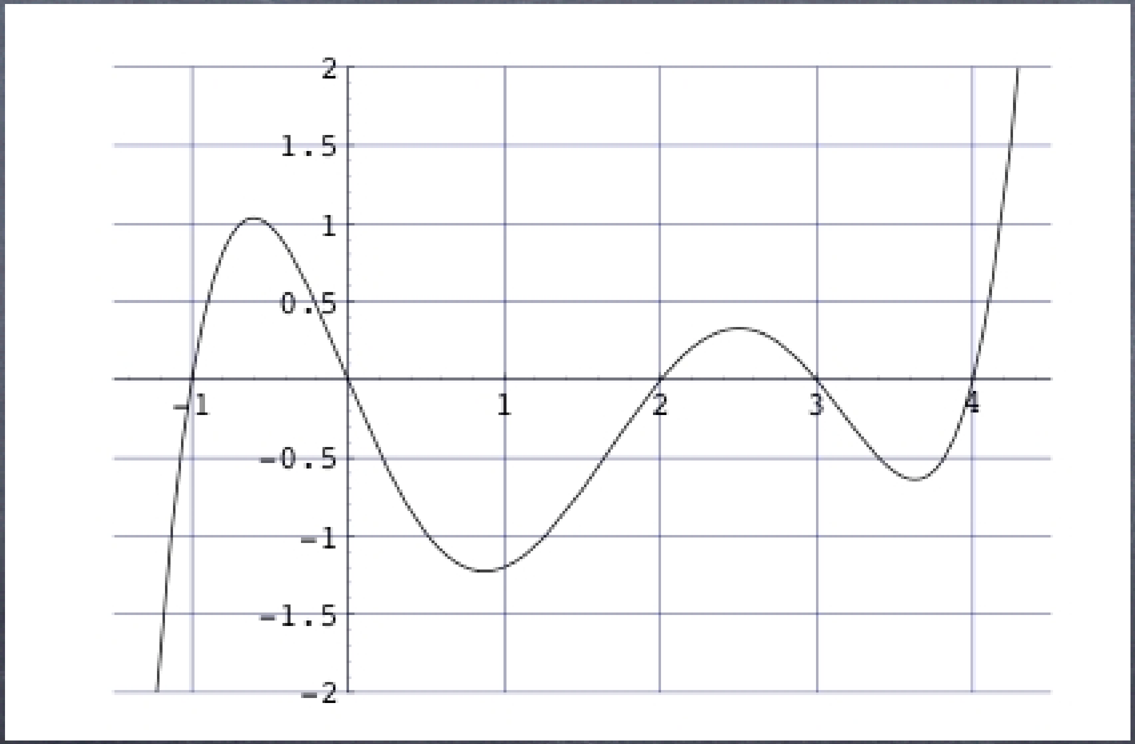
Two definitions of the derivative

- $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$
- $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$
- The derivative of $f(x) = \frac{1}{x} + \sqrt{x}$

Finding Limits



The graph of the derivative of f(x)

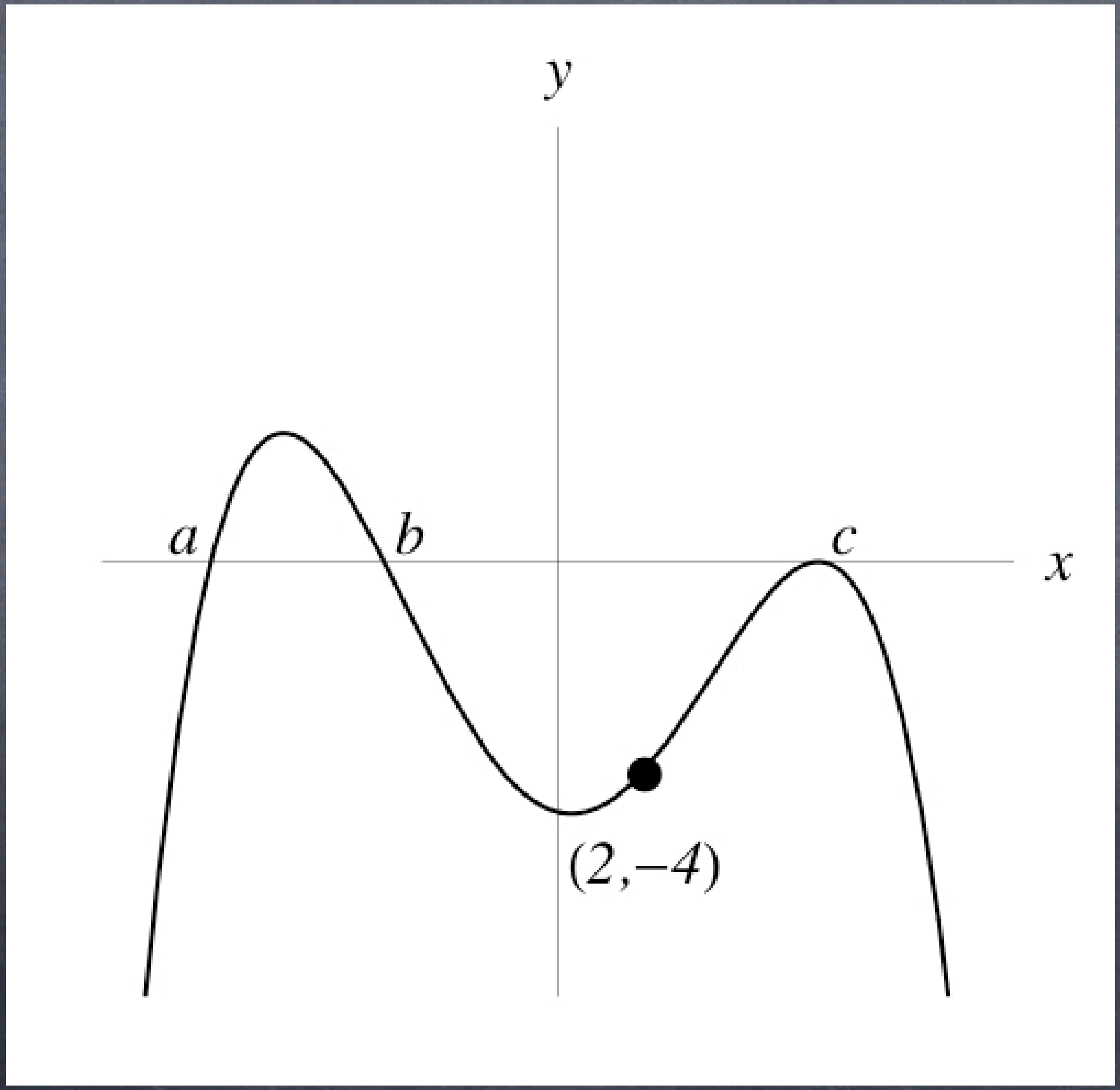


Sketching the graph of a function

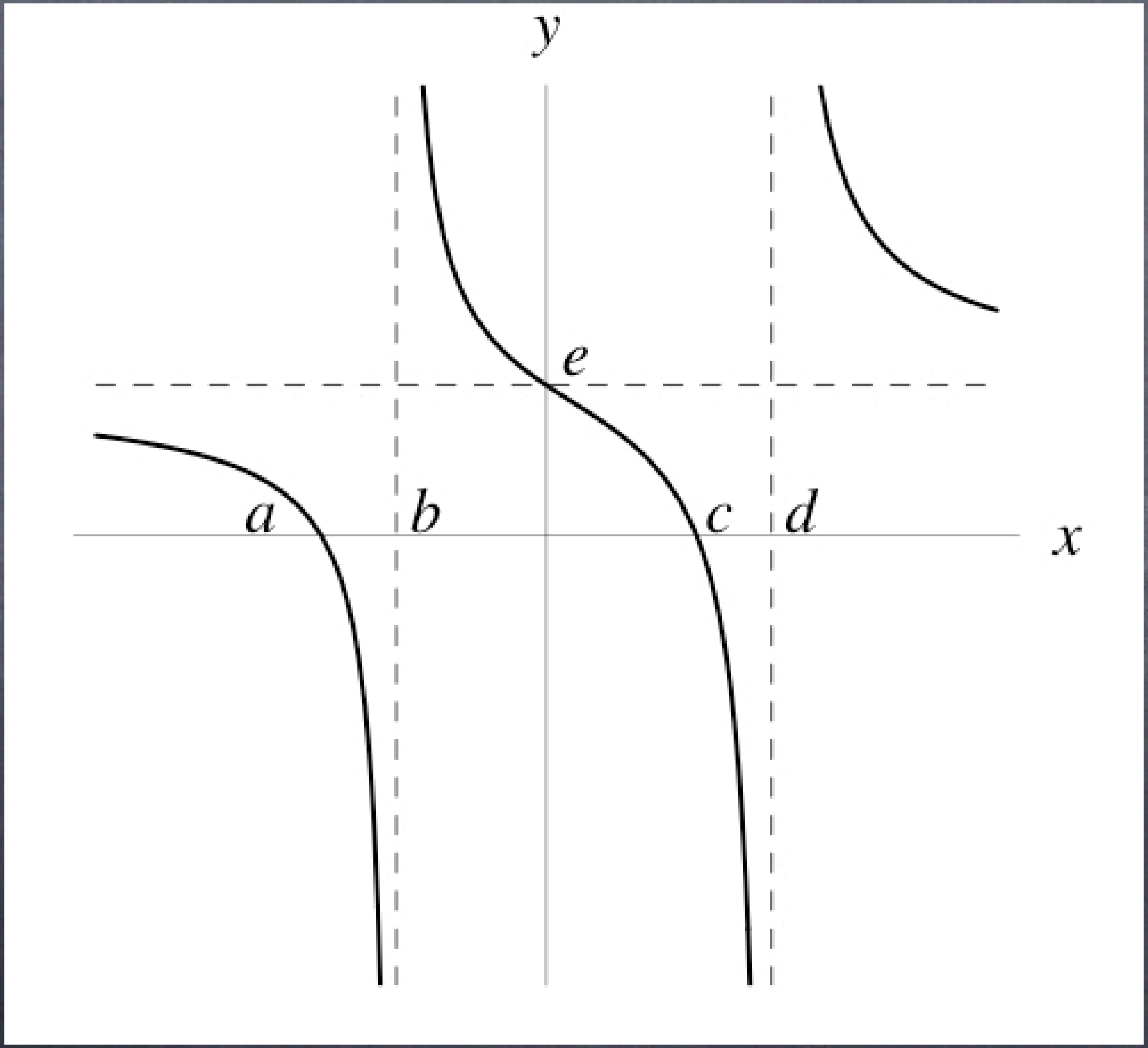
Assume that f is a continuous function on the closed interval $[-3, 3]$ with $f(-3) = 4$ and $f(3) = 1$. Also, assume that f' and f'' exist and are continuous on $(-3, 3)$. Use the information in the table below to sketch a possible graph of f.

x	$-3 \leq x < -1$	-1	$-1 < x < 0$	0	$0 < x < 1$	1	$1 < x \leq 3$
$f'(x)$	$+$	0	$-$	$-$	$-$	0	$-$
$f''(x)$	$-$	$-$	$-$	0	$+$	0	$-$

Finding a polynomial function



Finding a rational function



Estimation using linear approximation

When your morning coffee is poured, its temperature is 190°F. You sit down to enjoy your coffee in a nice 70°F room. After ten minutes the temperature of your coffee is 179°F and after ten minutes and six seconds the coffee has cooled to 177.9°F. Use linear approximation to estimate the temperature of your coffee after eleven minutes.

Exponents and exponential growth

A certain bacteria culture grows exponentially. The bacteria count was 400 after two hours and 25,600 after six hours.

(a) What was the initial population of the culture?

(b) Find an expression for the population after t hours.

(c) In what time period does the population double?

Optimization

- 1. Draw the picture
- 2. Name the variables
- 3. Write down the function to be optimized
- 4. Relate the variables

Optimization (continued)

- 5. Reduce the function to a function of one variable
- 6. Find the critical points
- 7. Apply the first or second derivative test
- 8. Write down the final answer

Running a trucking firm

A truck driving over a flat interstate at a constant rate of speed of 50 mph gets 4 miles to the gallon. Fuel costs a \$1.19 per gallon. The truck loses a tenth of a mile per gallon in fuel efficiency for each mile per hour increase in speed. Drivers are paid \$27.50 per hour in benefits and wages, and fixed costs for running the truck are \$11.33 per gallon. A trip of 300 miles is planned. What speed minimizes operating expenses?

Running a trucking firm (continued)

The total cost of operating the truck at s miles per hour for a 300 mile trip is

$$C(s) = \text{fixed costs} + \text{fuel costs} + \text{driver costs}$$

or

$$C(s) = \frac{300}{s} \cdot (11.33) + \frac{300}{4 - 0.1(s - 50)} \cdot (1.19) + \frac{300}{s} \cdot (27.50).$$

Solving the equation $C'(s) = 0$, tells us that a speed of $s \approx 57.930$ mph minimizes operating costs.