Math Xa
Review Session #3

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1/14/05

Last Third of the Class

- Inverse and one-to-one functions.
- Logarithms.
- The chain rule.
- Logarithmic differentiation.
- More optimization.

Inverse functions

- The horizontal line test
- One-to-one functions and $f'(x) > 0$
- Symmetric about $y = x$
- Finding inverse functions

Logarithms

- $\log_b xy = \log_b x + \log_b y.$
- $\log_b \dfrac{x}{y} = \log_b x - \log_b y.$
- $\log_b x^p = p \log_b x.$
- $\log_b b^x = x$ and $b^{\log_b x} = x.$
- $b^x = e^{\ln b^x} = e^{(\ln b)x}.$
- $\log_b 1 = 0.$
- $\log_b b = 1.$
- $\log_b x = \dfrac{\log_a x}{\log_a b}$

Derivatives of logs and exponentials

- $\frac{d}{dx}e^x = e^x$
- $\frac{d}{dx}b^x = b^x \ln b$
- $\frac{d}{dx}\ln x = \frac{1}{x}$
- $\frac{d}{dx}\log_b x = \frac{1}{x \ln b}$

The chain rule

- $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x).$
- $f(x) = x^{10}.$
- $f(x) = (3x^2 + 2x + 1)^{10}.$
- $f(x) = \ln(x^2 + 2x - 1).$
- $y = \sqrt{x^2 + e^{-x^2}}$

Logarithmic differentiation

$$f(x) = \ln\left(\frac{x^7(3x^3-2)^{10}}{\sqrt{x^2-3x+5}}\right)$$

Optimization

- 1. Draw the picture
- 2. Name the variables
- 3. Write down the function to be optimized
- 4. Relate the variables

Optimization (continued)

- 5. Reduce the function to a function of one variable
- 6. Find the critical points
- 7. Apply the first or second derivative test
- 8. Write down the final answer

Going to the Bus Stop

Susan wants to get to the bus stop as quickly as possible. The bus stop is across a grassy park, 2000 feet west and 600 feet north of her starting position. Susan can walk west along the side of the park on the sidewalk at a rate of 6 feet per second. She can also travel across the grass of the park, but only at a rate of 4 feet per second. What path will get her to the bus stop the fastest?

Going to the Bus Stop (continued)

The function to be maximized is

$$T = \frac{x}{6} + \frac{\sqrt{(2000 - x)^2 + 600^2}}{4}.$$

The critical point is $x \approx 1463$ feet, which gives us a time of about 445 seconds.