

1.2**Problem 14.**

(a) $f(0) = \sqrt{\frac{0}{0+1}} = 0$

(b) $f(3) = \sqrt{\frac{1}{3+1}} = \frac{1}{2}$

(c) $f(-\frac{1}{4}) = \sqrt{\frac{1}{-\frac{1}{4}+1}} = \sqrt{\frac{1}{\frac{3}{4}}} = \frac{2}{\sqrt{3}}$

(d) $f(b) = \sqrt{\frac{1}{b+1}}$

(e) $f(b-1) = \sqrt{\frac{1}{b-1+1}} = \frac{1}{\sqrt{b}}$

(f) $f(b+3) = \sqrt{\frac{1}{b+3+1}} = \frac{1}{\sqrt{b+4}}$

(g) $[f(7)]^2 = (\sqrt{\frac{1}{7+1}})^2 = \frac{1}{8}$

(h) $f(b^2) = \sqrt{\frac{1}{b^2+1}}$

(i) $[f(b)]^2 = (\sqrt{\frac{1}{b+1}})^2 = \frac{1}{b+1}$

Problem 16.

(a) $h(0) = \frac{0^2}{1-2(0)} = 0$

(b) $h(3) = \frac{3^2}{1-2(3)} = -\frac{9}{5}$

(c) $h(p+1) = \frac{(p+1)^2}{1-2(p+1)} = \frac{p^2+2p+1}{-2p-1}$

(d) $h(3p) = \frac{(3p)^2}{1-2(3p)} = \frac{9p^2}{1-6p}$

(e) $2h(3p) = 2(\frac{(3p)^2}{1-2(3p)}) = \frac{18p^2}{1-6p}$

(f) $\frac{1}{h(2p)} = \frac{1}{\frac{(2p)^2}{1-2(2p)}} = \frac{1-4p}{4p^2}$

Problem 17.

(a) 1

(b) 2

(c) 6

(d) $3x^2 + 2x + 1$

(e) $3x^2 + 10x + 9$

(f) $9x^2 - 6x + 3$

(g) $27x^2 - 6x + 1$

1.3**Problem 8.**

(a) $f(-1) = 0; f(0) = 2; f(1) = 3.$

(b) $x = -1, 2, 4$

(c) $x = 0, x \approx 1.6$

(d) $-f(0) + 2f(3) \approx -(2) + 2(-\frac{3}{4}) = -3.5$

Problem 13.

(a) $x = -3, x = 4$

(b) $-3 < x < 4$

Problem 29.

Let h be the height of the box. Then the volume of the box is given by $s^2h = 200$, and hence $h = \frac{200}{s^2}$.

The surface area of the box is given by $2s^2 + 4sh = 2s^2 + 4s \frac{200}{s^2} = 2s^2 + \frac{800}{s}$.

The total cost of the material for the box $C(s)$ is given by $C(s) = 2s^2 \cdot 10 + \frac{800}{s} \cdot 7 = 20s^2 + \frac{5600}{s}$.

Problem 43.

(a) Let C = the cost of the sculpture and w = the weight of the sculpture. Then

$$C = kw \quad \text{for some constant } k.$$

Validity: In general this would seem unlikely, since a much more important factor in the cost of a sculpture is the quality of the sculpture. However, this proportion may hold for works by a given artist.

(b) Let R = the rate at which money is growing and M = the amount of money. Then

$$R = kM \quad \text{for some constant } k.$$

Validity: This is generally true for a given account at a given bank, and the proportionality constant k is the interest rate.

- (c) Let R = the rate at which the population is growing and let P = the population. Then

$$R = kP \quad \text{for some constant } k.$$

Validity: This is generally true for a particular locale within certain bounds. The proportionality constant k represents the difference between the birth rate and the death rate.

- (d) Let d = the distance traveled and r = the rate at which you are traveling. Then

$$d = rt \quad \text{for some constant } t.$$

Validity: This is true whenever you are traveling at a constant rate. The constant t represents the time for which you are traveling.

Problem 55.

- (a) The volume of a cylinder of height h inches and base radius r inches is $V = \pi r^2 h$ cubic inches. Solving for h , we obtain $h = \frac{88}{\pi r^2}$.
- (b) Assume that the cylinder has height h inches and base radius r inches. The surface area of the cylinder is the sum of the areas the top and bottom circles and the side. Unwrapping the side yields a rectangle whose dimensions are h by $2\pi r$, which is the circumference of the circular base. Thus the surface area S is $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + (2\pi r) \frac{88}{\pi r^2} = 2\pi r^2 + \frac{176}{r}$ square inches.
- (c) Each circle requires $(2r)^2 = 4r^2$ square inches of cardboard, and the side requires $\frac{176}{r}$ square inches of cardboard. Hence the total amount of cardboard used is: $2(2r)^2 + \frac{176}{r} = 8r^2 + \frac{176}{r}$ square inches. As cardboard costs k cents per square inch, the cost of the material required for the container is $k(8r^2 + \frac{176}{r})$ cents.
- (d) The custom-made plastic lids and bottoms will each require $2\pi r^2$ square inches of plastic and will cost $7k$ cents per square inches. Hence the total cost of making the cylindrical container is $7k(2\pi r^2) + \frac{176k}{r} = 14\pi r^2 + \frac{176k}{r}$.

2.1

Problem 7.

- (a) $x = -1, x = 2$
(b) $(-1, 2)$
(c) $[-2, 1], [3, 4]$
(d) $[-2, -1), [3, 4)$

Problem 8.

- (a) $x = -1$ and $x = 4$.
(b) $(-1, 4)$
(c) $[-2, 3]$
(d) $[-2, -1)$

Problem 9.

- (a) $x = -2$.
(b) $(-2, 7]$
(c) $[-3, 1)$
(d) $[-3, -2)$

2.2

Problem 11.

(a) $[-5, 1]$: Analytical approach: Solve the equation $|x + 2| = h(x + 2) = 3$.

Case (1): $x + 2 \geq 0$	Case (2): $-(x + 2) < 0$
$x + 2 = 3$	$-(x + 2) = 3$
$x = 1$	$-x - 2 = 3$
	$-x = 5$
	$x = -5$

These two solutions partition the number line into three intervals: $(-\infty, -5)$, $(-5, 1)$, and $(1, \infty)$. Substituting the numbers -6 , 0 , and 2 , which respectively lie in these three intervals, into the original inequality, we obtain the solution set stated above.

Geometric approach: The symbolic statement " $h(x + 2) \leq 3$ " is equivalent to the statement " x is within 3 units from -2 , from which we obtain the solution set stated above.

(b) $x = -4$ or $x = 6$: Analytic approach: Solve the equation $|x - 1| = h(x - 1) = 5$.

Case (1): $x - 1 \geq 0$	Case (2): $x - 1 < 0$
$x - 1 = 5$	$-(x - 1) = 5$
$x = 6$	$-x + 1 = 5$
	$-x = 4$
	$x = -4$

Geometric approach: The symbolic statement " $h(x - 1) = 5$ " is equivalent to the statement " x is 5 units from 1, from which we obtain the solution stated above.

(c) $(-\infty, -3.1] \cup [-2.9, \infty)$: Analytical approach: Solve the equation $|x + 3| = h(x + 3) = 0.1$.

Case (1): $x + 3 \geq 0$	Case (2): $x + 3 < 0$
$x + 3 = 0.1$	$-(x + 3) = 0.1$
$x = -2.9$	$-x - 3 = 0.1$
	$-x = 3.1$
	$x = -3.1$

These two solutions partition the number line into three intervals: $(-\infty, -3.1)$, $(-3.1, -2.9)$, and $(-2.9, \infty)$. Substituting the numbers -4 , -3 , and 0 , which respectively lie in these three intervals,

into the original inequality, we obtain the solution set stated above.

Geometric approach: The symbolic statement " $h(x + 3) \geq 0.1$ " is equivalent to the statement " x is at least 0.1 unit from -3 ", from which we obtain the solution set stated above.

(d) $(-\infty, -\frac{5}{3}) \cup (-\frac{5}{3}, \infty)$: Analytical approach: Solve the equation $|3x + 1| = h(3x + 1) = 4$.

Case (1): $3x + 1 \geq 0$	Case (2): $3x + 1 < 0$
$3x + 1 = 4$	$-(3x + 1) = 4$
$3x = 3$	$-3x - 1 = 4$
$x = 1$	$-3x = 5$
	$x = -\frac{5}{3}$

These two solutions partition the number line into three intervals: $(-\infty, -\frac{5}{3})$, $(-\frac{5}{3}, 1)$, and $(1, \infty)$. Substituting the numbers -2 , 0 , and 2 , which respectively lie in these three intervals, into the original inequality, we obtain the solution set stated above.

Geometric approach: The symbolic statement " $h(3x + 1) > 4$ " is equivalent to the statement " $3x$ is greater than 4 units from -1 ", which is equivalent to the statement " x is greater than $\frac{4}{3}$ units from $-\frac{1}{3}$ ", from which we obtain the solution set stated above.

Problem 12.

(a) $2|x| > 4$ implies $|x| > 2$ and hence $x > 2$ or $x < -2$

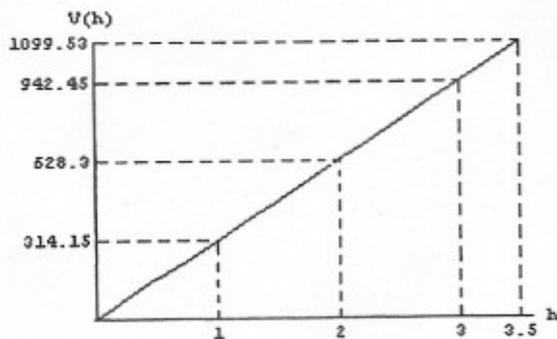
(b) $|2x - 1| \leq 3$ implies $-3 \leq 2x - 1 \leq 3$, this gives $-1 \leq x \leq 2$

(c) $|x^2 - 1| \geq 0$ gives x to be arbitrary number

Problem 13.

(a) The volume of a cylinder of height h feet and base radius r feet is $V = \pi r^2 h$ cubic feet. Here, $r = 10$ feet is constant, and h varies; hence $V(h) = 100\pi h$.

(b)



(c) $[0, 350\pi]$

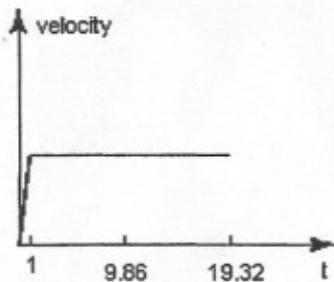
(d) The average rate of change of volume with respect to height, $\frac{\Delta V}{\Delta h} = 100\pi$, as the volume function is linear. Increasing the depth of water by 1 foot requires 100π additional cubic feet of water; a $1/2$ -foot increase requires 50π additional cubic feet of water.

(e) $\frac{\Delta V}{\Delta h} = 100\pi \text{ ft}^3/\text{ft} \approx (100\pi \text{ ft}^3/\text{ft})(1 \text{ gallon}/0.16043 \text{ ft}^3) \approx 1957 \text{ gallons}/\text{ft}$.

(f) Yes, the volume of water in the pool is directly proportional to the height of the water, and the proportionality constant is 100π .

Problem 14.

(a) Bailey's average speed was $\frac{100}{9.86} \approx 10.14$ m/sec, and Johnson's average speed was $\frac{200}{19.32} \approx 10.35$ m/sec. The 200 meter dash had the higher average speed, which was achieved by Johnson. Assume it took both runners the same amount of time, say 1 second, to achieve their top speeds, which were very close, from the start. In the 100 meter dash, Bailey is sprinting below his top speed for the first $1/9.86$ -th of the race, whereas Johnson is sprinting below his top speed for the first $1/19.86$ -th of the race. Hence Johnson ran at his top speed for a greater fraction of the duration of the 200 meter dash than Bailey ran at his top speed in the 100 meter dash. The graph below illustrates that Johnson was at his top speed for a greater fraction of the race.

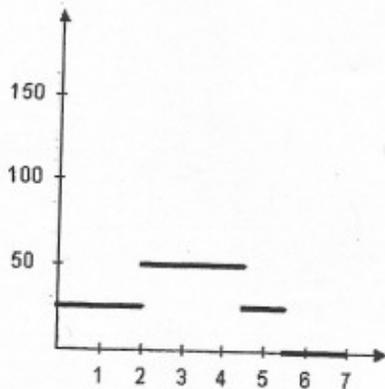


(b) No, the pattern does not hold. Reynolds's average speed was about 9.24 m/sec, which is slower than the speeds of Bailey and Johnson. The 400 meter dash is not a sprint in contrast to the 100 meter and 200 meter dashes. A runner's cruising speed in the 400 meter dash will not be his maximum speed, which is the case in the two sprints. Consequently, a runner in the 400 meter race will likely have a slower average speed than he would in the two sprints.

Problem 14.

2.4

- (a) $25 \frac{km}{h}$
- (b) 12:30 P.M.
- (c) go home
- (d) the graph is a line
- (e) $50 \frac{m}{h}$
- (f)



3.1

Problem 8.

(a) (i) $f(x) + g(x) = x(x+1) + x^3 + 2x^2 + x = x^2 + x + x^3 + 2x^2 + x = x^3 + 3x^2 + 2x$

(ii) $\frac{f(x)}{g(x)} = \frac{x(x+1)}{x^3+2x^2+x} = \frac{x(x+1)}{x(x^2+2x+1)} = \frac{x(x+1)}{x(x+1)^2} = \frac{1}{x+1}$

(iii) $\frac{g(x)}{f(x)} = \frac{x^3+2x^2+x}{x(x+1)} = \frac{x(x+1)^2}{x(x+1)} = x+1$

(iv) $\frac{[f(x)]^2}{g(x)} = \frac{[x(x+1)]^2}{x(x+1)^2} = \frac{x^2(x+1)^2}{x(x+1)^2} = x$

(b) $xf(x) = g(x) \Leftrightarrow x(x(x+1)) = x^3 + 2x^2 + x \Leftrightarrow x^2(x+1) = x(x+1)(x+1) \Leftrightarrow x^2(x+1) - x(x+1)^2 = 0$
 $\Leftrightarrow x(x+1)(x - (x+1)) = 0 \Leftrightarrow x(x+1)(-1) = 0 \Leftrightarrow x(x+1) = 0 \Leftrightarrow x = -1, 0.$

Problem 10.

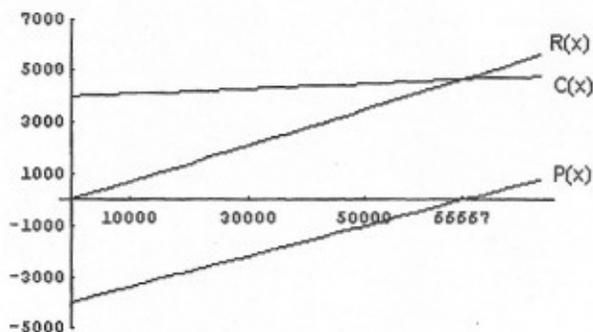
(a) $R(x) = 0.07 \cdot x$

(b) $C(x) = \$40000 + 0.01 \cdot x$

(c) $P(x) = R(x) - C(x) = 0.07 \cdot x - [4000 + 0.01 \cdot x] = 0.07 \cdot x - 4000 - 0.01 \cdot x = 0.06 \cdot x - 4000$

(d) Break even: $P(x) = 0$, so $0.06 \cdot x - 4000 = 0$, $0.06 \cdot x = 4000$, $x = \frac{4000}{0.06} = 66,666.7$ copies.

(e)

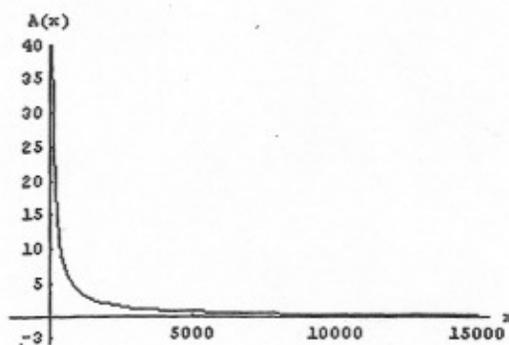


(f) $A(x) = \frac{\text{cost of copies}}{\text{number of copies}} = \frac{4000+0.01x}{x} = 0.01 + \frac{4000}{x}$

(g)

x	0	1	10	100	1000	10,000
$A(x)$	undefined	4000.01	400.01	40.01	4.01	0.41

(h)



3.2**Problem 5.**

- (a) $g(g(0)) = g(-3) \approx 2$
 (b) $f(f(0)) = f(1) = 0$

Problem 10.

$$f(0)x + f(1) = g(0)x + g(1) \Leftrightarrow (1)x + 0 = (-3)x + (-2) \Leftrightarrow x = -3x - 2 \Leftrightarrow 4x = -2 \Leftrightarrow x = -\frac{1}{2}.$$

Problem 19.

- (a) $f(-1)g(-1) = 2 \cdot 3 = 6$
 (b) $f(g(-1)) = f(3) = -2$
 (c) $g(f(-1)) = g(2) = 3$
 (d) $h(g(f(2))) = h(g(4)) = h(0) = -3$
 (e) $\frac{f(0)+2}{g(0)} = \frac{1+2}{4} = \frac{3}{4}$
 (f) $5h(3) + f(f(1)) = 5 \cdot 1 + f(3) = 5 + (-2) = 3$
 (g) $f(f(f(0))) = f(f(1)) = f(3) = -2$

Problem 20.

- (a) From the graph of f , we see that $f(x) = 0$ when $x = -2, 2$, or 4 . Now $g(x) = -2$ when $x = -3$; $g(x) = 2$ when $x \approx 1.5$ or $x \approx 2.9$; and $g(x)$ is never equal to 4 . Hence the zeros of $h(x)$ are $x = -3$, $x \approx 1.5$, and $x \approx 2.9$.
 (b) From the graph of g , we see that $g(x) = 0$ when $x = -2, 0$, or 4 . Now $f(x) = -2$ when $x = -3$ or $x = 3$; $f(x) = 0$ when $x = -2, 2$ or 4 ; and $f(x)$ is never equal to 4 . Hence the zeros of $j(x)$ are $x = -3$, $x = -2$, $x = 2$, $x = 3$, and $x = 4$.

3.3**Problem 10.**

$$h(x) = f(g(x)). \text{ Put } g(x) = x + 2, f(x) = \frac{3}{\sqrt{x}}$$

Problem 17.

$$h(x) = f(g(x)). \text{ Put } f(x) = 4x^2 + 3x + 2 \text{ and } g(x) = \pi^x.$$

3.4**Problem 5.**

- (a) (vi)
 (b) (ii)
 (c) (vii)
 (d) (viii)
 (e) (iii)
 (f) (i)

Problem 12.

- (a) Shift the graph of $y = x^2$ left 3 units, then flip over the x -axis, then shift down 1 unit.
 (b) Shift the graph of $y = x^2$ right 3 units, then up 1 unit.

Problem 16.

- (a) Shift the graph of $y = x^2$ right 2π units, then shift up π units.
 (b) Shift the graph of $y = x^2$ right 2π units, then flip over the x -axis, then shift up π units.

4.2**Problem 4.**

The point slope equation of a line is $y - y_1 = m(x - x_1)$. That is $y - (-3) = -\frac{1}{2}(x - (-2)) \Leftrightarrow y = -\frac{1}{2}x - 4$.

Problem 13.

The equation vertical line passing through the point (a, b) has the form $x = a$. Hence the equation of the vertical line passing through $(-\sqrt{\pi}, \pi^2)$ is $x = -\sqrt{\pi}$.

4.3

Problem 2.

Solve $1.1x = \$1000$, this gives $x = \frac{\$1000}{1.1} \approx \909

Problem 8.

$$y = \frac{9}{2}x + 9$$

4.4

Problem 1.

Let $W(x)$ be the social worker's weekly wages as a function of the number of hours worked, x .

$$W(x) = \begin{cases} Dx & \text{for } 0 \leq x \leq 40 \\ 40D + 1.5D(x - 40) & \text{for } x > 40, \end{cases}$$

Problem 2.

- (a) Let y be the price of a container of yogurt, and x the price of a packet of honey. From the given information, we have the following system of equations in x and y :

$$180,000 = y + 2x$$

$$310,000 = 2y + 3x.$$

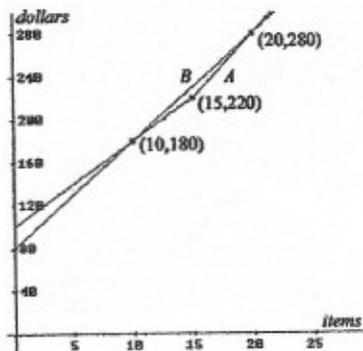
We multiply the first equation by 2 to obtain $360,000 = 2y + 4x$ and then subtract the second equation from this equation to see that $x = 50,000$. Substituting this value for x into the first equation, we see that $y = 180,000 - 2(50,000) = 80,000$. The price of a yogurt is 80,000 lira, and 50,000 is the price of a packet of honey.

Problem 5.

(a)

item	0	5	10	15	20
A	100	140	180	220	280
B	80	130	180	230	280

- (b) By graphing the functions representing both salary schemes on the same axes, we see that Company B is better for sales between 10 and 20 items per week.



(c)

$$S_A(x) = \begin{cases} 100 + 8x & \text{for } 0 \leq x \leq 15 \\ 220 + 12(x - 15) = 40 + 12x & \text{for } x > 15 \end{cases}$$

$$S_B(x) = 80 + 10x$$

To determine when $S_B(x) > S_A(x)$, first solve $80 + 10x > 100 + 8x$ for $0 \leq x \leq 15$: we get $x > 10$; hence, B is better for $10 < x \leq 15$. Now solve $80 + 10x > 40 + 12x$ for $x > 15$; we get $x < 20$, so B is also better for $15 < x < 20$. Combining, $S_B(x) > S_A(x)$ is better for $10 < x < 20$.