Problem 1. 5.1

pg 1

(a) By completing the square, we see that $h(t) = -16t^2 + 8t + 48 = -16\left(t^2 - \frac{1}{2}t\right) + 48 = -16\left(t - \frac{1}{4}\right)^2 + 49$. The graph of h is a parabola that opens downward with its vertex at $\left(\frac{1}{4}, 49\right)$. Therefore, since $1 > \frac{1}{4}$, the ball is heading down at $t = 1$.

(b) The average rate of change of height with respect to time over $[0.9, 1] = \frac{\Delta h}{\Delta t} = \frac{h(1)-h(0.9)}{1-0.9} = \frac{-16(1)^2+8(1)+48-(-16(0.9)^2+8(0.9)+48)}{0.1} = -22.4$ ft/sec.
The average rate of change of height with respect to time over $[1, 1.1] = \frac{\Delta h}{\Delta t} = \frac{h(1.1)-h(1)}{1.1-1} = \frac{-16(1.1)^2+8(1.1)+48-(-16(1)^2+8(1)+48)}{0.1} = -25.6$ ft/sec.

The ball's velocity is between -25.6 ft/sec and -22.4 ft/sec.

(c) The average rate of change of height with respect to time over $[0.99, 1] = \frac{\Delta h}{\Delta t} = \frac{h(1)-h(0.99)}{1-0.99} = \frac{-16(1)^2+8(1)+48-(-16(0.99)^2+8(0.99)+48)}{0.01} = -23.84$ ft/sec.
The average rate of change of height with respect to time over $[1, 1.01] = \frac{\Delta h}{\Delta t} = \frac{h(1.01)-h(1)}{1.01-1} = \frac{-16(1.01)^2+8(1.01)+48-(-16(1)^2+8(1)+48)}{0.01} = -24.16$ ft/sec.

The ball's velocity is between -24.16 ft/sec and -23.84 ft/sec.

(d) $h'(1) = \lim_{\Delta t\to 0} \frac{h(1+\Delta t)-h(1)}{\Delta t} = \lim_{\Delta t\to 0} \frac{-16(1+\Delta t)^2+8(1+\Delta t)+48-(-16(1)^2+8(1)+48)}{\Delta t} = \lim_{\Delta t\to 0} \frac{-32\Delta t-16(\Delta t)^2+8\Delta t}{\Delta t} = \lim_{\Delta t\to 0} -16\Delta t - 24 = -24$. The instantaneous velocity of the ball at $t = 1$ is -24 ft/sec.

Problem 4.

(a) The slope of the secant line through P and $Q = m(h) = \frac{f(0+h)-f(0)}{h} = \frac{h^3}{h} = h^2$. Now $m(-0.1) = 0.01$, $m(-0.01) = 0.0001$, $m(-0.001) = 0.000001$, $m(0.0001) = 0.00000001$, $m(0.001) = 0.000001$, $m(0.01) = 0.0001$, and $m(0.1) = 0.01$.

(b) It appears that $f'(0) = 0$.

(c) $f'(0) = \lim_{h\to 0} m(h) = \lim_{h\to 0} h^2 = 0$.

(d) The function $f(x) = x^3$ is increasing everywhere, which implies that any difference quotient will be positive. As $f'(0) = 0$, these difference quotients are certainly greater than $f'(0)$.

Problem 6.
$f'(4) = -\frac{1}{16} = 0.625$

$4+h$	3.9	3.99	3.999	4.0001	4.001	4.01	4.1
$\frac{f(4+h)-f(4)}{h}$	-0.06370	-0.06262	-0.06251	-0.06250	-0.06249	-0.06238	-0.06135

Problem 17.

(a) $A = (4-w, g(4-w))$, $B = (4, g(4))$, $C = (4+w, g(4+w))$, $D = (s, g(s))$, $E = (s+p, g(s+p))$, $F = (r, g(r))$

(b) i) B, ii) C, iii) E, iv) A, v) G, vi) F, vii) D, viii) G

5.2

Problem 2.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{k(x+h)^2 - kx^2}{h} = \lim_{h\to 0} \frac{k[x^2+2xh+h^2-x^2]}{h}$$
$$= \lim_{h\to 0} \frac{k(2xh+h^2)}{h} = \lim_{h\to 0} \frac{kh(2x+h)}{h} = 2kx$$

Problem 7.
From problem 1 we have $f'(x) = 3$. $f'(0) = 3$; $f'(2) = 3$; $f'(-1) = 3$.

pg2

Problem 16.

(a) $\dfrac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x} = \dfrac{\frac{1}{(x+\Delta x)^2}-\frac{1}{x^2}}{\Delta x} = \dfrac{-2x\Delta x-(\Delta x)^2}{(x+\Delta x)^2x^2\Delta x} = \dfrac{-2x-\Delta x}{(x+\Delta x)^2x^2}.$

(b) $f'(x) = \lim\limits_{\Delta x\to 0}\dfrac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x} = \lim\limits_{\Delta x\to 0}\dfrac{-2x-\Delta x}{(x+\Delta x)^2x^2} = -\dfrac{2}{x^3}.$

5.3

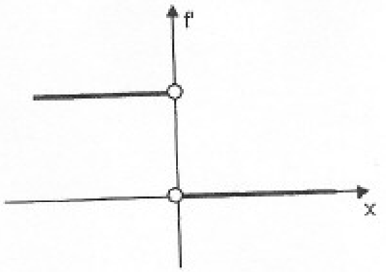
Problem 2.

(a) $[2,6]$

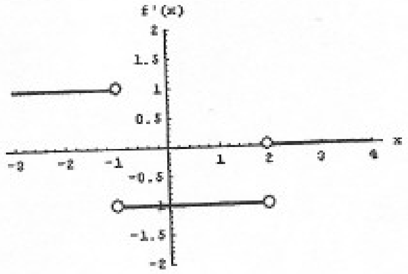
(b) $[0,2]$

(c) $x=2$

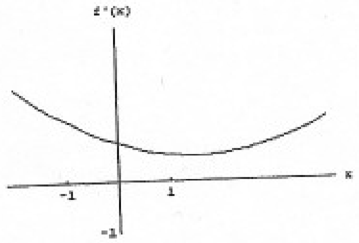
Problem 5.



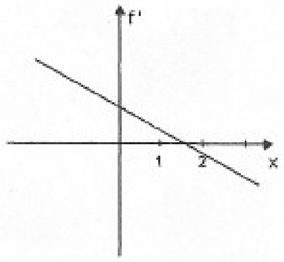
Problem 6.



Problem 7.



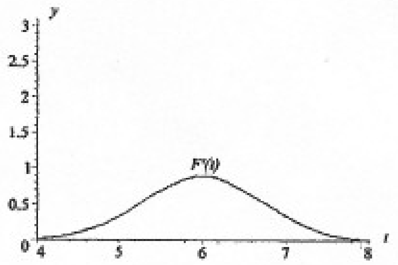
Problem 8.



5.4

Problem 3.

(a)



(b) The slope of F is the rate of change of the number of fish in the pond over time; the slope decreases over time as $F(t)$ approaches C.

Problem 4.

(a) Decrease in food by one unit will result in increase in clothing by two units.

(b) Very small decrease in food corresponds to very large increase in clothing, then quality of food is small.

6.1

Problem 4.

$y' = -x+1$; $0 = y' = -x+1 \Rightarrow x = 1$ and $y(1) = -\frac{2(1)^2+7}{4} + \frac{3(1)-1}{3} = -\frac{19}{12}$. As the coefficient of x^2 is negative, the vertex $\left(1, -\frac{19}{12}\right)$ is the highest point on the curve.

Problem 5.

(a) The function is $f(x) = 3x + C$, where C is a constant. If $f(x)$ passes through the origin then we have $0 = C$, and hence $f(x) = 3x$. If $f(x)$ passes through $(0,2)$ then we have $2 = C$, and $f(x) = 3x+2$.

(b) The function is $f(x) = \pi x + C$. If it passes through the origin, then $C = 0$ and $f(x) = \pi x$. If it passes through $(0,2)$ then $C = 2$ and $f(x) = \pi x + 2$.

6.2

Problem 1.

(a) $h(x) = f(x)g(x) = (x+3)(x-5) = 0 \Rightarrow x = -3$ or $x = 5$.

(b) $h(x) = (x+3)(x-5) = -7 \Leftrightarrow 0 = x^2 - 2x - 8 = (x+2)(x-4) \Rightarrow x = -2$, or $x = 4$.

(c) $h(x) = (x+3)(x-5) = -15 \Leftrightarrow 0 = x^2 - 2x = x(x-2) \Rightarrow x = 0$, or $x = 2$.

(d) $h(x) = (x+3)(x-5) = c \Leftrightarrow 0 = x^2 - 2x - 15 - c$ Using the quadratic formula to solve this last equation for x, we have $x = \frac{-(-2)\pm\sqrt{(-2)^2-4(-15-c)}}{2} = 1 \pm \sqrt{16+c}$.

(e) $j(x^2) - 2 = \frac{f(x^2)}{g(x^2)} - 2 = \frac{x^2+3}{x^2-5} - 2 = 0. \Leftrightarrow 0 = \frac{x^2+3-2(x^2-5)}{x^2-5} = \frac{-x^2+13}{x^2-5} \Leftrightarrow -x^2+13 = 0$. Now $-x^2+13 = 0$ $\Rightarrow x = \pm\sqrt{13}$. Since $(\pm\sqrt{13})^2 - 5 = 13 - 5 = 8 \neq 0$, the solutions are $x = \pm\sqrt{13}$.

(f) $[j(x)]^2 - 1 = \left[\frac{f(x)}{g(x)}\right]^2 - 1 = \frac{(x+3)^2}{(x-5)^2} - 1 = \frac{(x+3)^2-(x-5)^2}{(x-5)^2} = \frac{16x-16}{(x-5)^2} = 0 \Leftrightarrow 16x - 16 = 0$. Now $16x - 16 = 0$ when $x = 1$, and $(1-5)^2 = 16 \neq 0$. Hence the solution to the equation is $x = 1$.

(g) $h(x) = j(x) \Leftrightarrow f(x)g(x) = \frac{f(x)}{g(x)} \Leftrightarrow (x+3)(x-5) = \frac{x+3}{x-5} \Leftrightarrow \frac{(x+3)((x-5)^2-1)}{x-5} = 0$. Now $0 = (x+3)((x-5)^2-1) = (x+3)(x-4)(x-6) \Rightarrow x = -3$, $x = 4$, or $x = 6$. Notice that none of these three values of x make the denominator from the original equation, $(x-5)$, equal to zero. Hence $x = -3$, $x = 4$, and $x = 6$ are the solutions to the equation.

Problem 2.

(a) (*vii*);

(b) (*vi*);

(c) (*iii*);

(d) (*viii*);

(e) (*i*);

Problem 3.

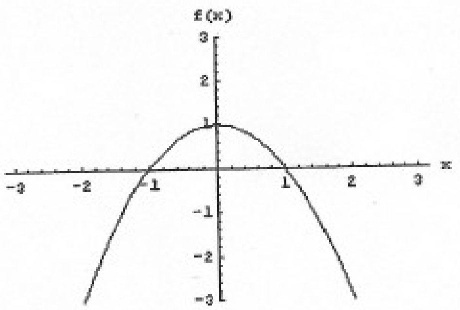
(a) $x^2 - 7 = 0$ gives $x^2 = 7, x = \pm\sqrt{7}$

(c) $(x+1)^2 = 25, x+1 = \pm 5, x = -6$ or $x = 4$

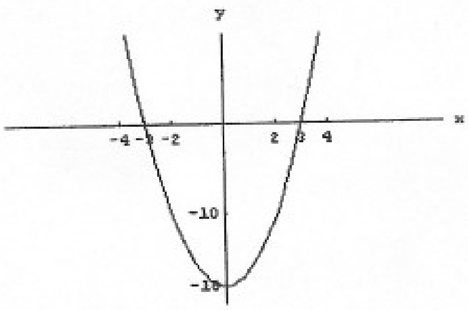
(g) $x = -3, x = 1$

6.3

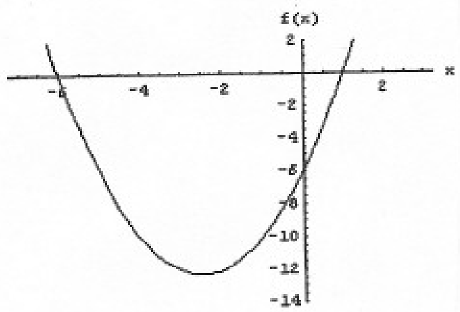
Problem 1.



Problem 2.

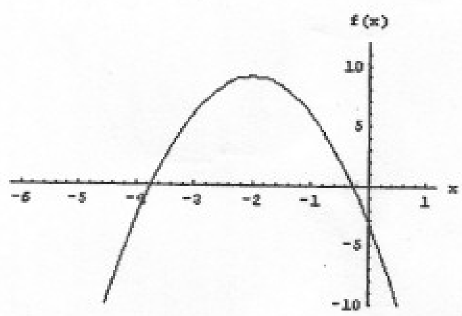


Problem 3.

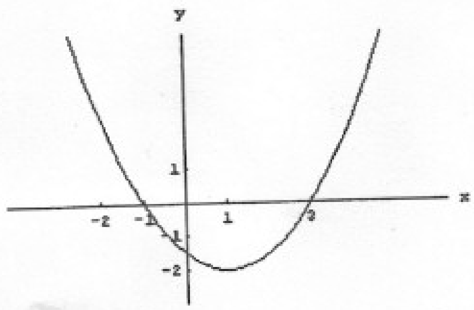


(4)

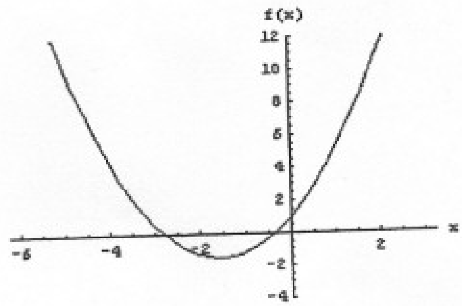
Problem 6.



Problem 7.



Problem 8.



Problem 9.

(a) Let $y = k(x+4)(x-2)$; as $(-1,2)$ is on the graph, $2 = k(-1+4)(-1-2)$, from which we obtain $k = -\frac{2}{9}$. Therefore, $y = -\frac{2}{9}(x+4)(x-2)$.

(b) Let $y = k(x+2)(x-1)$; as $(0,-3)$ is on the graph, $-3 = k(0+2)(0-1)$, from which we obtain $k = \frac{3}{2}$. Therefore, $y = \frac{3}{2}(x+2)(x-1)$

(c) Let $y = k(x-3)^2$; as $(0,-3)$ in on the graph, $-3 = k(0-3)^2$, from which we obtain $k = -\frac{1}{3}$. Therefore, $y = -\frac{1}{3}(x-3)^2$

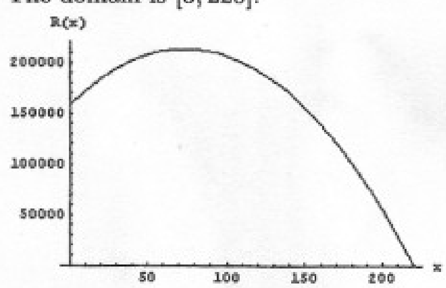
(d) Let $y = k(x+2)(x-1)(x-3)$; as $(0,-2)$ is on the graph, $-2 = k(0+2)(0-1)(0-3)$, from which we obtain $k = -\frac{1}{3}$. Therefore, $y = -\frac{1}{3}(x+2)(x-1)(x-3)$.

6.4

Problem 1.

(a) $R(x) = (720 + 10x)(220 - x) = -10x^2 + 1480x + 158,400$

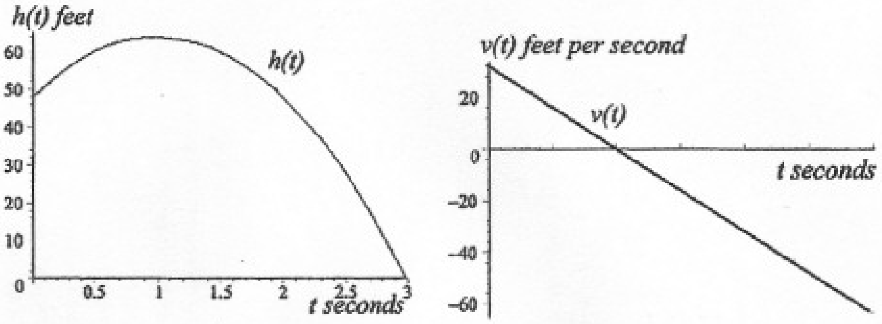
(b) The domain is $[0, 220]$.



(c) As R is a quadratic function with negative lead coefficient, its graph is a parabola that opens downward. The maximum value of the revenue function R is the $y-$coordinate of the vertex of the parabola. The $x-$coordinate of the vertex is the number of unsold seats that will result in the maximum revenue. We find the $x-$coordinate of the vertex by solving the equation $0 = R'(x) = -20x + 1480$. This solution to this equation is $x = 74$, and $R(74) = -10(74)^2 + 1480(74) + 158,400 = 213,160$. Therefore, 74 unsold seats result in the maximum profit of \$213,160.

Problem 3.

(a)



(b) The ball was thrown from a height of $h(0) = 48$ feet.

(c) $v(t) = h'(t) = -32t + 32$; $v(0) = 32$. The initial velocity was 32 ft/sec. The ball was thrown up because $v(0) = 32$ is positive.

(d) The ball's height was decreasing at $t = 2$ because its velocity at $t = 2$, $v(2) = -32(2) + 32 = -32$, is negative.

(e) The ball achieves its maximum height when $v(t) = -32(t) + 32 = 0$, which is when $t = 1$. That is, the ball achieves its maximum height 1 second after being thrown. At $t = 1$, the height of the ball is $h(1) = -16(1)^2 + 32(1) + 48 = 64$ feet, and the velocity is $v(1) = 0$ ft/sec.

(f) The ball hits ground at the first instance after $t > 0$ when $h(t) = -16(t + 1)(t - 3) = 0$, which is when $t = 3$. Therefore the ball was in the air for 3 seconds.

(g) The ball's acceleration was $a(t) = v'(t) = -32$ ft/sec^2. Yes; this makes physical senses because acceleration of a falling body due to gravity is -32 ft/sec^2.

Problem 9.

(a) Amelia sells $q(x) = 120 - 5x$ bowls per week at a price of x dollar per bowl. Now $q(x + 1) - q(x) = (120 - 5(x + 1)) - (120 - 5x) = -5$. By raising the price of a bowl by 1 dollar, she sells 5 fewer bowls.

(b) $R(x) = (\text{price})(\text{quantity}) = xq(x) = x(120 - 5x)$.

(c) As her revenue function R is quadratic, her revenue will be maximzed at the sales level of x bowls for which $R'(x) = 0$. Now $R'(x) = 120 - 10x$, and hence $R'(x) = 0$ when $x = 12$. Therefore her revenue is maximized when she charges \$12 per bowl.

(d) The maximum weekly revenue is $R(12) = \$720$.

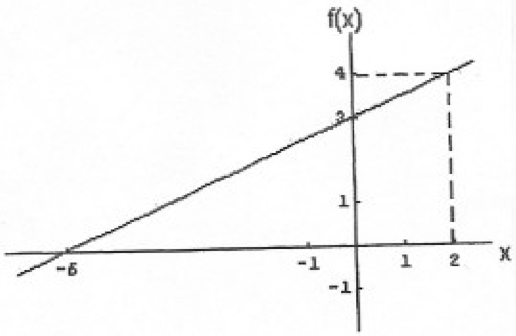
7.1

Problem 2.

(a) $\lim_{n\to\infty} \left(-\frac{1}{2}\right)^n = 0$

(b) Statements (iii) and (v) are true. The limit $\lim_{n\to\infty} (-2)^n$ does not exist as the values oscillate between large positive and large negative numbers. We also have that $\lim_{n\to\infty} -2^n = -\lim_{n\to\infty} 2^n = -\infty$.

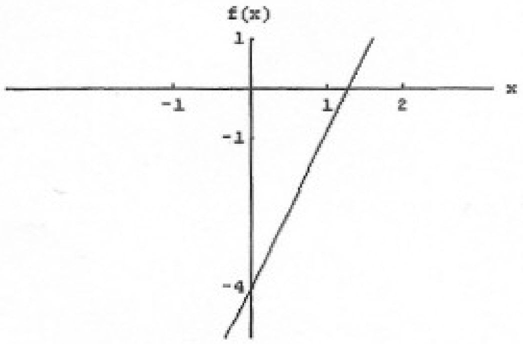
Problem 5.



$f(x) = \frac{x}{2} + 3$, $\lim_{x\to 2} f(x) = 4$

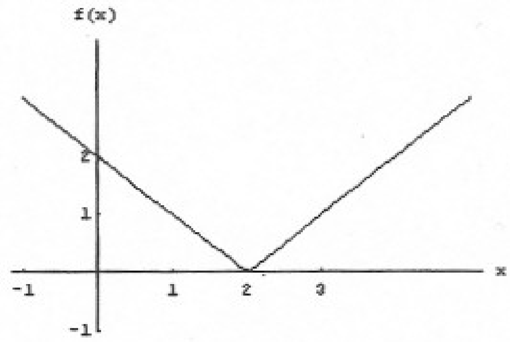
Problem 6.

$f(x) = \pi x - 4$
(a) $\lim_{x \to 0} f(x) = -4$
(b) $\lim_{x \to 1} f(x) = \pi - 4 \approx -.86$
(c) $\lim_{x \to \infty} f(x) = \infty$



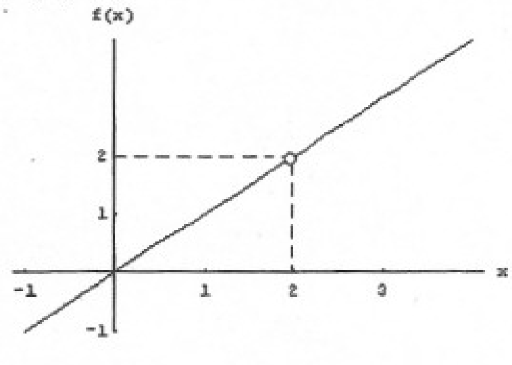
Problem 7.

$f(x) = |x - 2|$
(a) $\lim_{x \to 0} f(x) = 2$
(b) $\lim_{x \to 2} f(x) = 0$



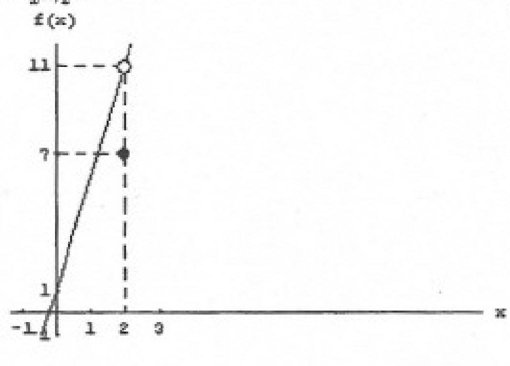
Problem 8.

$f(x) = \frac{x^2-2x}{x-2} = \frac{x(x-2)}{x-2} = x,\ D := x \neq 2$
(a) $\lim_{x \to 0} f(x) = 0$
(b) $\lim_{x \to 2} f(x) = 2$



Problem 9.

$f(x) = \begin{cases} 5x+1 & ,x \neq 2 \\ 7 & ,x = 2 \end{cases}$
(a) $\lim_{x \to 1} f(x) = 6$
(b) $\lim_{x \to 2} f(x) = 11$



pg 7

Problem 17.
$f(x) = \sqrt{7+x}$, $a = 0$, $f'(0) = \lim_{h\to 0} \frac{\sqrt{7+h}-\sqrt{7}}{h} = \lim_{h\to 0} \frac{(\sqrt{7+h}-\sqrt{7})(\sqrt{7+h}+\sqrt{7})}{h(\sqrt{7+h}+\sqrt{7})} = \lim_{h\to 0} \frac{7+h-7}{h(\sqrt{7+h}+\sqrt{7})} =$
$\lim_{h\to 0} \frac{1}{\sqrt{7+h}+\sqrt{7}} = \frac{1}{2\sqrt{7}}$

7.2

Problem 2.

(a) $\lim_{x\to 2^-} \frac{1}{x-2} = -\infty$

(b) $\lim_{x\to 2^+} \frac{1}{x-2} = +\infty$

(c) $\lim_{x\to 2} \frac{1}{x-2} = DNE$, because the left- and right-hand limits are not equal.

Problem 3.

(a) $\lim_{x\to 1^-} \frac{1}{(x-1)^2} = \infty$

(b) $\lim_{x\to 1^+} \frac{1}{(x-1)^2} = \infty$

(c) $\lim_{x\to 1} \frac{1}{(x-1)^2} = \infty$

(d) $\lim_{x\to -1} \frac{1}{(x-1)^2} = \frac{1}{4}$

Problem 4.

$$f(x) = \frac{|x-3|}{x-3} = \begin{cases} \frac{x-3}{x-3} = 1 & \text{if } x > 3 \\ \frac{-(x-3)}{x-3} = -1 & \text{if } x < 3 \\ undefined & \text{if } x = 3 \end{cases}$$

(a) $\lim_{x\to 0} f(x) = -1$

(b) $\lim_{x\to 4} f(x) = 1$

(c) $\lim_{x\to 3^+} f(x) = 1$

(d) $\lim_{x\to 3^-} f(x) = -1$

(e) $\lim_{x\to 3} f(x) = DNE$

Problem 6.

(a) $\lim_{x\to \frac{1}{\pi}} f(x) = 2.$

(b) $\lim_{x\to -\frac{1}{\pi}} f(x) = -2.$

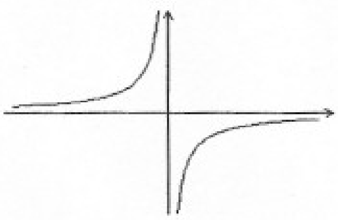
(c) $\lim_{x\to 0} f(x)$ does not exist because $\lim_{x\to 0^-} f(x) = -1$ and $\lim_{x\to 0^+} f(x) = 1$

Problem 16.
Let $f(x) = \begin{cases} x-3 & , x \neq 5 \\ 7 & , x = 5 \end{cases}$
$\lim_{x\to 5} f(x) = 2$ but $f(x) = 7$

7.4

Problem 1.

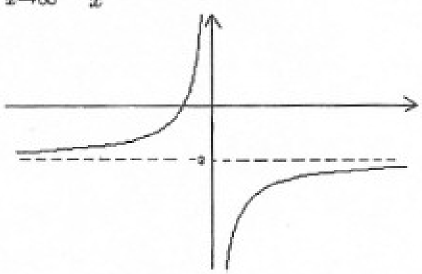
(a)



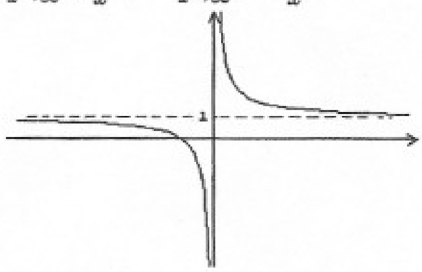
(i) $\lim_{x\to -\infty} (-\frac{3}{x}) = 0$

(ii) $\lim_{x\to \infty} (-\frac{3}{x}) = 0$

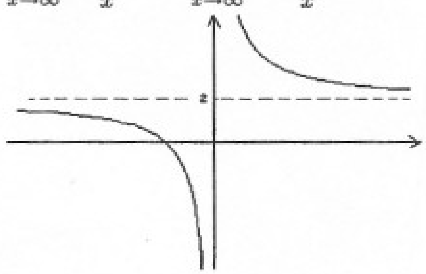
(iii) $\lim_{x\to \infty} (-\frac{3}{x} - 3) = -3$



(iv) $\lim_{x\to \infty} (\frac{x+1}{x}) = \lim_{x\to \infty} (1 + \frac{1}{x}) = 1$



(v) $\lim_{x\to \infty} (\frac{2x+3}{x}) = \lim_{x\to \infty} (2 + \frac{3}{x}) = 2$



(b) (i) $\lim_{x\to-\infty}(-\frac{3}{x}) = \lim_{x\to\infty}(\frac{3}{x}) = \lim_{x\to\infty}(3)\cdot\lim_{x\to\infty}(\frac{1}{x}) = 3\cdot 0 = 0$

(ii) $\lim_{x\to\infty}(-\frac{3}{x}) = \lim_{x\to\infty}(-3)\cdot\lim_{x\to\infty}(\frac{1}{x}) = -3\cdot 0 = 0$

(iii) $\lim_{x\to\infty}(-\frac{3}{x}-3) = \lim_{x\to\infty}(-3)\cdot\lim_{x\to\infty}(\frac{1}{x}) - \lim_{x\to\infty}(3) = -3\cdot 0 - 3 = -3$

(iv) $\lim_{x\to\infty}(\frac{x+1}{x}) = \lim_{x\to\infty}(1+\frac{1}{x}) = \lim_{x\to\infty}(1) + \lim_{x\to\infty}(\frac{1}{x}) = 1+0 = 1$

(v) $\lim_{x\to\infty}(\frac{2x+3}{x}) = \lim_{x\to\infty}(2+\frac{3}{x}) = \lim_{x\to\infty}(2) + \lim_{x\to\infty}(3)\cdot\lim_{x\to\infty}(\frac{1}{x}) = 2+3\cdot 0 = 2$

Problem 3.

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4}\frac{\sqrt{x}-2}{(\sqrt{x}-2)(\sqrt{x}+2)} = \frac{1}{4}$$

Problem 5.

$f(x) = \frac{x^2-4}{x+2} = \frac{(x-2)(x+2)}{x+2} = x-2,\ D_f: x \neq -2.$

$x = -2$ is a removable point of discontinuity. We can define a function at $x = -2$ to make it continuous at -2. For example let $f(x) = \begin{cases} \frac{x^2-4}{x+2} & ,x \neq -2 \\ -4 & ,x = -2 \end{cases}$

Problem 6.

There is a removable point of discontinuity at $x = 0$; it can be removed by defining $f(0) = 0$.

Problem 17.

(a) Yes

(b) No

8.1

Problem 2.

(a) To approximate $\sqrt{102}$ use the graph of $\sqrt{x}$ and its tangent line at $x = 100$, then $\sqrt{102} \approx 10 + 0.05(102 - 100)$

(b) $f(x) = \sqrt{x}$ at $x = 9$; $\sqrt{8} \approx 3 + \frac{1}{6}(8-9)$

(c) $f(x) = \sqrt{x}$ at $x = 16$; $\sqrt{18} \approx 4 + \frac{1}{8}(18-16)$

(d) $f(x) = \sqrt{x}$ at $x = 121$; $\sqrt{115.5} \approx 11 + \frac{1}{22}(115.5-121)$

Problem 5.

$f(x) = \sqrt[3]{x},\ x = 27.$

The slope of the tangent to $f(x)$ is $(\frac{1}{3}x^{-\frac{2}{3}})|_{x=27} = \frac{1}{27}$. So, the tangent is $y - 3 = \frac{1}{27}(x-27)$ or $y = 3 + \frac{1}{27}(x-27)$.

Hence $\sqrt[3]{30} \approx 3 + \frac{1}{27}(30-27) \approx 3.111$

Using calculator we check that $\sqrt[3]{30} \approx 3.107$

8.3

Problem 1.

$f'(x) = 6x + 3 - 3x^{-2} - 6x^{-3}$

Problem 2.

$f(x) = \frac{1}{5}(x - 2x^2) \Rightarrow f'(x) = \frac{1}{5}(1-4x)$

Problem 3.

Product Rule. $f'(x) = \pi[(6x+7)(x-2) + (3x^2+7x+1)] = \pi[6x^2 - 12x + 7x - 14 + 3x^2 + 7x + 1] = \pi[9x^2 + 2x - 13]$

Problem 4.

Quotient Rule. $f'(x) = \frac{0\cdot(x^2+4) - 1(2x)}{(x^2+4)^2} = \frac{-2x}{(x^2+4)^2}$

Problem 5.

Quotient Rule. $f'(x) = \frac{(x+2)(1) - (x)(1)}{(x+2)^2} = \frac{2}{(x+2)^2}$

Problem 6.

$f(x) = \frac{x+2}{x} = 1 + 2x^{-1},\quad f'(x) = -2x^{-2} = \frac{-2}{x^2}$

pg 9

Problem 7.

$f(x) = \left(\frac{5}{2}x^2 + 7x^5 - 5x\right)x = \frac{5}{2}x^3 + 7x^6 - 5x^2 \Rightarrow f'(x) = \frac{15}{2}x^2 + 42x^5 - 10x.$

Problem 8.

$f(x) = ax^{-1} + bcx - bdx^2, \quad f'(x) = -ax^{-2} + bc - 2bdx = -\frac{a}{x^2} + b(c - 2dx)$

Problem 9.

(a) $f(x) = \frac{x^3+2x^2}{3} = \frac{1}{3}(x^3 + 2x^2), \quad f'(x) = \frac{1}{3}(3x^2 + 4x)$

(b) $f(x) = \frac{(x^2+1)^2}{x} = \frac{x^4+2x^2+1}{x} = x^3 + 2x + x^{-1}, \quad f'(x) = 3x^2 + 2 - x^{-2} = 3x^2 + 2 - \frac{1}{x^2}$

Problem 10.

(a) $f(x) = \frac{\frac{1+x}{x}}{\frac{x+1}{x(x+2)}} = \frac{1+x}{x} \cdot \frac{x(x+2)}{x+1} = x + 2, \quad f'(x) = 1$

(b) $f'(x) = -\frac{1}{2x^2} - \frac{9}{2x^4}$

Problem 11.

(a) $f'(x) = 3x^2 - 1$

(b) $f'(x) = -\frac{2}{x^3}$

9.1

Problem 2.

(a) Let $G(s)$ = the number of grains as a function of the s^{th} square. Thus $G(s) = 2^{s-1}$, and hence $G(64) = 2^{63} \approx 9.22 \times 10^{18}$ grains of rice were allocated to the 64th square.

(b) Let $W(s)$ = weight of rice at square s measured in grams. Thus $W(s) = 0.02G(s) = 0.02(2^{s-1})$, and hence $W(64) = 0.02(2^{63}) \approx 0.02(9.22 \times 10^{18}) \approx 1.844 \times 10^{17}$ grams, which is about $(1.844 \times 10^{17})/(907.18 \times 1000) \approx 2.03 \times 10^{11}$ tons of rice. This is about 500 times the world production of rice in 1980.

(c)

squares		total grains
1		$1 = 2^1 - 1$
2	$1+2=$	$3 = 2^2 - 1$
3	$1+2+4=$	$7 = 2^3 - 1$
4	$1+2+4+8=$	$15 = 2^4 - 1$
.		.
.		.
64		$2^{64} - 1 \approx 1.84 \times 10^{19}$ grains

So the total mass is about 3.69×10^{17} grams.

9.2

Problem 32.

(a) $\frac{x^{2y}+x^{y+2}}{x^y} = \frac{x^y(x^y+x^2)}{x^y} = x^y + x^2$

(b) $-\frac{1}{x+y}$

(c) $\frac{A^{B+4}-A^{3B}}{A^B(A^2-A^B)} = \frac{A^B(A^4-A^{2B})}{A^B(A^2-A^B)} = \frac{(A^2)^2-(A^B)^2}{(A^2-A^B)} = \frac{(A^2-A^B)(A^2+A^B)}{(A^2-A^B)} = A^2 + A^B$

(d) $\frac{y^w(y^{2w}-y^4)}{y^w(y^w+y^2)} = \frac{(y^w-y^2)(y^w+y^2)}{(y^w+y^2)} = y^w - y^2$

Problem 34.

(a) 0

(b) 0

Problem 35.

(a) -1

(b) ∞

Problem 36.

(a) ∞

(b) 0

Problem 37.

(a) 7

(b) $-\infty$

Problem 2.

Substituting $t = 149$ into the formula, $C(t) = C_0\left(\frac{1}{2}\right)^{t/5730}$., obtained in problem 1(a), we have: $C(149) = C_0\left(\frac{1}{2}\right)^{149/5730} \approx C_0(0.982137)$. Hence, 98.2% of the original C_{14} remained in 1999.

Problem 4.

(a) If the population was increasing linearly, then population $P(t)$, where t is measured in years after 1970, has the form $P(t) = \left(\dfrac{200,000 - 100,000}{20 - 0}\right) t + 100,000 = 5000t + 100,000$. In 1980, the population would be equal to $P(10) = 5000(10) + 1000,000 = 150,000$ people.

(b) If the population was increasing exponentially, then $P(t)$ has the form $P(t) = 100,000a^t$. Now $P(20) = 200,000 \Rightarrow 200,000 = 1000a^20 \Rightarrow a = 2^{1/20}$. Hence $P(10) = 100,000(2^{10/20}) \approx 141,421 \leq 141,422$ people, which is less than 150,000 people.

Problem 8.

Since we have an interval of 12 to 16 hours, we need to make two formulas: one for the slowest growth scenario, and one for the quickest.

Let $L(t)$ be the number of bacteria after t hours, assuming a 12-hour doubling time. Then $L(t) == 10 \cdot 2^{t/12}$. Now $1000 = L(t) = 10 \cdot 2^{t/12} \Rightarrow 100 = 2^{t/12} \Rightarrow \ln 100 = \ln(2^{t/12}) \Rightarrow \ln 100 = \left(\frac{t}{12}\right)\ln 2. \Rightarrow t = \frac{12 \ln 100}{\ln 2} \approx 79.73$.

Let $M(t)$ be the number of bacteria after t hours, assuming a 12-hour doubling time. Then $M(t) == 10 \cdot 2^{t/16}$. Now $1000 = L(t) = 10 \cdot 2^{t/16} \Rightarrow 100 = 2^{t/16} \Rightarrow \ln 100 = \ln(2^{t/16}) \Rightarrow \ln 100 = \left(\frac{t}{16}\right)\ln 2. \Rightarrow t = \frac{16 \ln 100}{\ln 2} \approx 106.30$.

If we begin with 10 bacteria that double in number every 12 to 16 hours, we can expect to see a 1000 bacteria in anytime from 79.7 to 106.3 hours. (Both values for t above can be obtained using a graphing calculator.)

9.4

Problem 12.

(a) $f'(x) = \frac{2xe^x + x^2e^x}{3} = \frac{e^x}{3}(x^2 + 2x)$

(b) $f'(x) = \frac{5}{3}\left(\frac{2x - x^2}{e^x}\right)$

(c) $f'(x) = \frac{-e^{5x} - 5xe^{5x}}{(xe^{5x})^2} = \frac{e^{5x}(-1-5x)}{x^2e^{10x}} = \frac{-1-5x}{x^2e^{5x}}$

Problem 15.

(a) Rewrite $f(x)$ as x^2e^{-x}, then using the Product Rule $f'(x) = 2xe^{-x} - x^2e^{-x} = \frac{2x - x^2}{e^x}$

(b) $f'(x) > 0$ if $\frac{2x - x^2}{e^x} > 0$. Since $e^x > 0$ for all x's it is enough to solve $2x - x^2 > 0$ which is $x(2 - x) > 0$, so $0 < x < 2$. Thus $f'(x)$ is positive if $0 < x < 2$ and $f'(x)$ is negative if $x < 0$ and $x > 2$.

(c) f is increasing for $x \in (0, 2)$. $f(x)$ is decreasing for $x \in (-\infty, 0)$ and $x \in (2, \infty)$

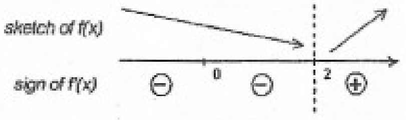
(d) The smallest value of $f(x)$ is $f(0) = 0$, since $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = \infty$.

10.1

Problem 10.

(a) $f'(x) = 12x^3 - 24x^2 = 12x^2(x - 2) \Rightarrow 0 = f'(x) = 12x^2(x - 2) \Rightarrow x = 0$ $x = 2$ are the critical points.

(b)



The first derivative test implies that $x = 0$ is neither a local maximum nor a local minimum point and $x = 2$ is a local minimum point. Moreover, as $x = 0$ is not an endpoint of the domain, it is not an absolute maximum or minimum point As $f(x)$ increases without bound as $|x|$ increases without bound, $x = 2$ is also the absolute minimum point.

(c) The absolute minimum value is $f(2) = -13$, and there is no absolute maximum value.

Problem 11.

(a) From problem 10, $x = 0$ is a critical point. The endpoints of the domain $[-1, 1]$, $x = -1$ and $x = 1$ are also critical points.

(b) From problem 10, $x = 0$ is neither a local maximum or minimum point nor an absolute maximum or minimum point. The point $x = -1$ is the absolute maximum point. $x = -1$ is the absolute minimum point.

(c) The absolute minimum is $f(1) = -2$, and the absolute maximum value is $f(-1) = 14$.

Problem 12.

(a) From problem 10, $x = 2$ is the only critical point.

(b) From problem 10, $x = 2$ is a local and absolute minimum point.

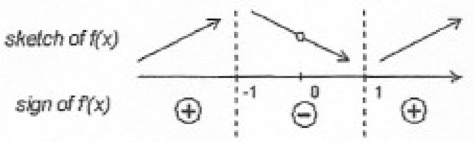
(c) The absolute minimum value is $f(2) = -13$, and there is no absolute maximum value.

Problem 13.

Note that 0 is not in the natural domain of $f(x)$ since $f(0)$ is undefined.

(a) $f'(x) = x^2 + 2 - \frac{3}{x^2} = \frac{x^4 + 2x^2 - 3}{x^2} = \frac{(x^2+3)(x+1)(x-1)}{x^2} \Rightarrow 0 = f'(x) \Rightarrow x = -1$ are critical points $x = 1$. The point $x = 0$ is not a critical point, because 0 is not in the domain of f.

(b)



The first derivative test implies that $x = -1$ is a local maximum point and $x = 1$ is a local minimum point. As $f(x)$ increases without bound as as x increases without bound, $x = -1$ is not an absolute maximum point. As $f(x)$ decreases without bound as as x decreases without bound, $x = 1$ is not an absolute minimum point.

(c) There are no absolute minimum or maximum values.

Problem 14.

(a) From problem 13, $x = -1$ is a critical point. The endpoint $x = -3$ of the domain is also a critical point.

(b) The point $x = -3$ is neither a local maximum or minimum point nor an absolute maximum or minimum point. From problem 13, $x = -1$ is a local maximum point and is also the absolute maximum point.

(c) There is no absolute minimum value, and the absolute maximum value is $f(-1) = -\frac{16}{3}$.

Problem 15.

(a) From problem 13, $x = 1$ is a critical point. The endpoint $x = 3$ of the domain $(0, 3]$ is also a critical point.

(b) From problem 13, $x = 1$ is a local minimum point and is also the absolute minimum point. The point $x = 3$ is neither a local maximum or minimum point nor an absolute maximum or minimum point.

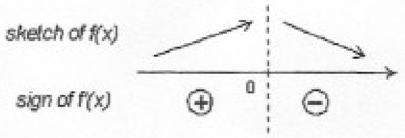
(c) The absolute minimum value is $f(1) = \frac{16}{3}$, and there is no absolute maximum value.

pg 12

Problem 16.

(a) $f'(x) = \frac{(x^2+4)(0)-(2x)(1)}{(x^2+4)^2} = -\frac{2x}{(x^2+4)^2} \Rightarrow 0 = f'(x) \Rightarrow x = 0$ is the only critical point.

(b)



The first derivative test implies $x = 0$ is a local and absolute maximum point.

(c) The absolute maximum value is $f(0) = \frac{1}{4}$ and there is no absolute minimum value.

10.2

Problem 3.

(a) $f'(x) = 3x^2 + 9x - 12 = 3(x+4)(x-1)$; $0 = f'(x) \Rightarrow x = -4$ and $x = 1$ are the critical points.

(b) $f''(x) = 6x + 9$. As $f''(-4) < 0$, $x = -4$ is a local maximum point; as $f''(1) > 0$, $x = 1$ is a local minimum point.

Problem 4.

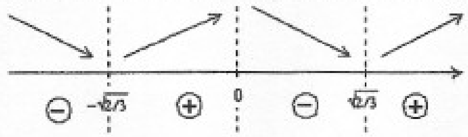
(a) $f'(x) = 5x^4 - 5 = 5(x^4 - 1) = 0$, so $x = \pm 1$ are critical points.

(b) $f''(x) = 20x^3$. Since $f''(-1) = -20 < 0$, f has a local maximum at $x = -1$. Since $f''(1) = 20 > 0$ f has a local minimum at $x = 1$

Problem 6.

(a) $f'(x) = 6x^5 - 4x^3 = 2(3x^5 - 2x^3) = 2x^3(3x^2 - 2) = 0$, so $x = 0$ and $x = \pm\sqrt{\frac{2}{3}}$ are critical points.

(b) $f''(x) = 30x^4 - 12x^2$. Since $f''(0) = 0$ the second derivative test fails. Alternative method is to look at



the sign of f'.
Hence at $x = \pm\sqrt{\frac{2}{3}}$ f has a local minimums and at $x = 0$ f has a local maximum.

Problem 7.

(a) $f'(x) = 4x^3 + 12x^2 = 4x^2(x+3)$; $0 = f'(x) \Rightarrow x = -3$ and $x = 0$ are the critical points.

(b) $f''(x) = 12x^2 + 24x$. As, $f''(-3) > 0$, $x = -3$ is a local minimum point. As $f''(0) = 0$, the second derivative test does not apply. Moreover, as $f'(x)$ is positive on $(-3,0) \cup (0,\infty)$, $x = 0$ is neither a local maximum nor a local minimum point.

Problem 8.
Domain $x \neq 0$.
(a) $f'(x) = \frac{-4+4x^3}{x^2}$; $f'(x) = 0 \Rightarrow 4x^3 - 4 = 0 \Rightarrow x^3 = 1 \Rightarrow x = 1$ is a critical point

(b) From Second Derivative Test we conclude that $x = 1$ is a local minimum $[f''(1) > 0]$

Problem 9.

(a) $f'(x) = e^x - 1 = 0$, so $e^x = 1$ and $x = 0$.

(b) Second Derivative Test $f''(x) = e^x$. Since $f''(0) = e^0 = 1 > 0$, f has a local minimum at $x = 0$.

Problem 11.

(a) $f'(x) = x^4 - 4x^3 + 4x^2$; $f'(x) = 0 \Leftrightarrow x^2(x^2 - 4x + 4) = 0$. So, the critical points are $x = 0$ and $x = 2$.

(b) Second Derivative Test fails ($f''(0) = 0$). By First Derivative Test there is no local extremum (f' is always non-negative)

Problem 12.

(a) $f'(x) = 12x^3 - 24x^2 + 6 = 6(2x^3 - 4x^2 + 1)$; $0 = f'(x) \Rightarrow x \approx -0.452$, $x \approx 0.597$, and $x \approx 1.855$ are the critical points.

(b) $f''(x) = 36x^2 - 48x$. $f''(-0.452) > 0$, $f''(0.597) < 0$, and $f''(1.855) > 0$, hence we have local minimum, maximum, and minimum points, respectively.

10.3

Problem 1.
The critical points occur at the values of x for which $f'(x) = \frac{e^x(x-1)^2}{(x^2+1)^2}$ is zero or undefined; hence $x = 1$ is the only critical point. As $f'(x) \geq 0$, for all x's, $x = 1$ is neither a local maximum nor a local minimum point. f has no absolute extrema.

Problem 3.
Again let l feet and w feet be the dimensions of the 90-square foot garden. Now $lw = 90 \Rightarrow l = \frac{90}{w}$, and we proceed to minimize the perimeter function $P(w) = 2l + 2w = 2(\frac{90}{w}) + 2w$ on $(0, \infty)$. We calculate $P'(w) = -\frac{180}{w^2} + 2$; hence, $P'(w) = 0$ when $w = \sqrt{90} = 3\sqrt{10}$. As $P''(w) = \frac{360}{w^3} > 0$ for all $w > 0$, $P(3\sqrt{10})$ is indeed the minimum value of P on $(0, \infty)$. Now $w = 3\sqrt{10} \Rightarrow l = \frac{90}{3\sqrt{10}} = 3\sqrt{10}$, and therefore, the plot is a square.

Problem 6.

(a) critical points: $-5, -1.5, 2, 4, 5$

(b) $x = 2$

(c) $x = 4$

Problem 7.

(i) $g(x) = 2f(x)$
   - (a) stretching vertically by a factor of 2
   - (b) we can obtain the new derivative from the old one by stretching it by a factor of 2.
   - (c) local minimum at $x = -1$; all points in $(0, 1]$

(ii)
   - (a) shifting down three units to obtain the graph of j.
   - (b) the graph of j' is the same as the graph of f'.
   - (c) local minimum at $x = -1$, and $(0, 1]$

(iii)
   - (a) Where the graph of f lies above or on the x-axis the graph of m is identical. Where the graph of f lies below the x-axis the graph of m is obtained by flipping the graph of f over the x-axis.
   - (b) On interval $(0, 4)$ the graph of m' is the same as f'. On interval $(-2, 0)$ the graph of m' is obtained by flipping the graph of f' over the x-axis.
   - (c) $[0, 1]$ - all points in.

(iv)
   - (a) The graph of f is shifted two units to the right to obtain the graph of k.
   - (b) The graph of f' is shifted two units to the right to obtain the graph of k'.
   - (c) $x = 1$, and all points in $(2, 3]$

(v)
   - (a) The graph of f is horizontally compressed by a factor of 2 to obtain the graph of h.
   - (b) The graph of f is horizontally compressed by a factor of 2 and then vertically stretched by a factor of 2 to obtain the graph of h'.
   - (c) $x = -\frac{1}{2}$, and all points in $(0, \frac{1}{2}]$

Problem 14.

The cost of the can is $C = k(2\pi rh) + 3k((2)(2r)^2)$, where k is a constant. As the volume of the can is $V = \pi r^2 h = 250$ cm^3, we have that $h = \frac{250}{\pi r^2}$. As a function of the radius r, the cost can be expressed as $C(r) = 2k\pi r(\frac{250}{\pi r^2}) + 24kr^2 = \frac{500k}{r} + 24kr^2$. Now $C'(r) = -\frac{500k}{r^2} + 48kr$, and thus $C'(r) = 0$ when $r = \sqrt[3]{\frac{125}{12}} \approx 2.18$ cm. We see that $C''(r) = \frac{500k}{r^3} + 48k > 0$ for all $r > 0$, which implies that the minimum cost is achieved when $r = \sqrt[3]{\frac{125}{12}} = \frac{5}{\sqrt[3]{12}} \approx 2.18$ cm. If $r = \frac{5}{\sqrt[3]{12}}$, $h = \dfrac{250}{\pi\left(\frac{5}{\sqrt[3]{12}}\right)^2} = \dfrac{20\sqrt[3]{18}}{\pi} \approx 16.68$.

Therefore the dimensions of the optimal can are a base radius of 2.18 cm and a height of 16.68 cm.

Problem 20.

(a) $P = 2(10 + 2y) + 2(8 + 2x) = 36 + 4(x + y)$.

(b) The area of the matting material is $A = (10 + 2y)(8 + 2x) - (10)(8) = 20x + 16y + 4xy = 200$ square inches, from which we obtain $y = \frac{50-5x}{4+x}$ inches. As a function of x, we have $P(x) = 36 + 4(x + \frac{50-5x}{4+x})$.

(c) $P'(x) = 4 - \frac{280}{(4+x)^2}$. $P'(x) = 0 \Rightarrow x^2 + 8x - 54 = 0 \Rightarrow x = \frac{-8+\sqrt{280}}{2} = -4 + \sqrt{70} \approx 4.37$ inches. Now $P''(x) = \frac{560}{(4+x)^3} > 0$ for all $x > 0$. Hence $= -4 + \sqrt{70}$ maximizes $P(x)$. If $x = -4 + \sqrt{70}$ and $y = \frac{200-20(-4+\sqrt{70})}{16+4(-4+\sqrt{70})} =\approx 3.37$. The dimensions of the frame that minimize cost are $10 + 2y = 16.74$ inches by $8 + 2x = 16.74$ inches.

11.1

Problem 21.

$f(x) = -2x^3 + 3x^2 + 6x - 2 \Rightarrow f'(x) = -6x^2 + 6x + 6 \Rightarrow f''(x) = -12x + 6 \Rightarrow$ The point of inflection is $(\frac{1}{2}, f(\frac{1}{2})) = (\frac{1}{2}, \frac{3}{2})$. Now $f'(\frac{1}{2}) = \frac{15}{2}$. The equation of the tangent line is $(y - \frac{3}{2}) = \frac{15}{2}(x - \frac{1}{2}) \Leftrightarrow y = \frac{15}{2}x - \frac{9}{4}$.

14-20 on Next page

pg 15

Problem 14.

$f(x) = x^3 + x^2 + x + 1 \Rightarrow f'(x) = 3x^2 + 2x + 1 = 0$. Now $0 = f'(x) \Rightarrow x = \frac{-2 \pm \sqrt{-8}}{6}$. Hence there are no real values of x for which $0 = f'(x)$ and consequently no critical points.

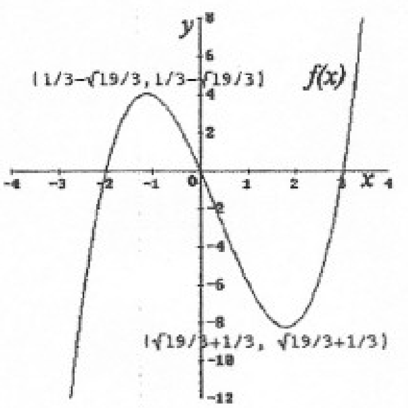
Problem 15.

$f(x) = -2x^3 + x^2 + 7 \Rightarrow f'(x) = -6x^2 + 2x = -2x(3x - 1)$. Now $0 = f'(x) \Rightarrow x = 0$ or $x = \frac{1}{3}$. We compute $f''(x) = -12x + 2$ and see that $f''(0) = 2 > 0$ and that $f''\left(\frac{1}{3}\right) = -2 < 0$. Hence there is a local minimum at $x = 0$ and a local maximum at $x = \frac{1}{3}$. As $\lim_{x \to \infty} f(x) = -\infty$ and $\lim_{x \to -\infty} f(x) = \infty$, neither of the local extrema are absolute extrema.

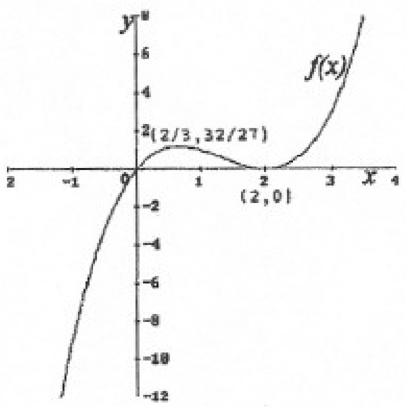
Problem 16.

$f(x) = x^3 + 2x^2 + 3x + 4 \Rightarrow f'(x) = 3x^2 + 4x + 3$. Now $0 = f'(x) \Rightarrow x = \frac{-4 \pm \sqrt{4^2 - 4(3)(3)}}{6}$. Hence there are no real values of x for which $0 = f'(x)$ and consequently no critical points.

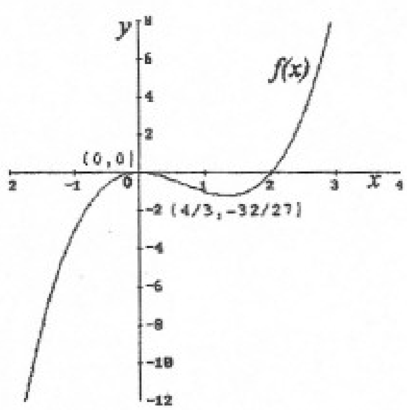
Problem 17.



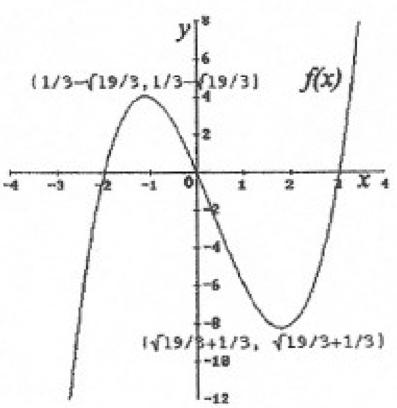
Problem 18.



Problem 19.



Problem 20.



Problem 10.

$P(x) = kx^2(x-\pi)^2(x+2)$, for some $k \neq 0$. As $\lim_{x\to\infty} P(x) = \infty$, k can be any positive number, and hence the answer is not unique.

Problem 11.

$P(x) = k(x+1)^3$, for some $k \neq 0$. As $\lim_{x\to\infty} P(x) = \infty$, k can be any positive number, and hence the answer is not unique.

Problem 12.

$P(x) = a(x-\pi)^2 + 2$, for some $a \neq 0$. As $0 = P(0) = a(0-\pi)^2 + 2$, $a = -\frac{2}{\pi^2}$. This answer is unique.

Problem 14.

$P(x) = k(x-1)^2(2-x)$, for some $k \neq 0$. Now $\sqrt{e} = P(0) = k(0-1)^2(2-0) \Rightarrow k = \frac{\sqrt{e}}{2}$. This answer is unique.

Problem 16.

(a) $f(x) = -x^3 - x^2 - 5x = -x(x^2 + x + 5)$. Now if $x^2 + x + 5 = 0$, then $x = \frac{-1\pm\sqrt{1-4(5)}}{2}$, which are not real solutions. The only zero is $x = 0$.

(b) $g(x) = 0.5x^4 - 0.5 = 0.5(x^2+1)(x-1)(x+1)$. Now $x^2 + 1 = 0$ has no real solutions; hence, the zeros are $x = -1$ and $x = 1$.

Problem 19.

$P(x) = x^4 - 2x^3 - 6x^2 + 12x = x(x-2)(x^2-6) = x(x-2)(x-\sqrt{6})(x+\sqrt{6})$. The zeros are $x = 0, 2, \sqrt{6}$, and $-\sqrt{6}$.

Problem 20.

$g(x) = 3x^3 + 3 = 3(x+1)(x^2 - x + 1)$. Now, as $(-1)^2 - 4(1)(1) < 0$, $x^2 - x + 1 = 0$ has no real solutions. Hence, the zero is $x = -1$.

11.3

Problem 8.

The strategy here should be to use the x−intercepts to establish the factors of the appropriate polynomial and then to use the additional point provided to determine the multiplicative factor.

(a) The zeros of the function are $x = -2, 1$, and 2. As the graph has the shape of a graph of a cubic polynomial, the function could have an equation of the form $P(x) = k(x+2)(x-1)(x-2)$, where k is a constant. Because the graph has a y−intercept of 3, we have $3 = P(0) = k(0+2)(0-1)(0-2) \Rightarrow k = \frac{3}{4}$. Therefore, $P(x) = \frac{3}{4}(x+2)(x-1)(x-2)$.

(b) This graph is the reflection of the graph in part (a) in the x−axis. Thus, $P(x) = -\frac{3}{4}(x+2)(x-1)(x-2)$.

(c) This function has an even order zero at $x = -2$ and an odd order zero at $x = 0$. As the graph has the shape of the graph of a cubic polynomial, the function could have an equation of the form $P(x) = kx(x+2)^2$. Because the graph contains the point $(1, 2)$, we have $2 = P(1) = k(1)(1+2)^2 = 9k \Rightarrow k = \frac{2}{9}$. Therefore, $P(x) = \frac{2}{9}x(x+2)^2$

Problem 12.

(a) (i) True.

(ii) False; $P(x) = (x-1)(x-2)(x-3)(x-4)(x-5)$ has 5 zeros.

(iii) False; $P(x) = x^5 + x$ has derivative $P'(x) = x^4 + 1$ which is positive for all x, forcing $P(x)$ to have no turning points.

(iv) True.

(b) Statement (i) is true. Because $P'(\pi) = 0$ and $P''(\pi) > 0$, $P(x)$ has a local minimum at $x = \pi$; because $P(x)$ has degree 5, it cannot have an absolute minimum.

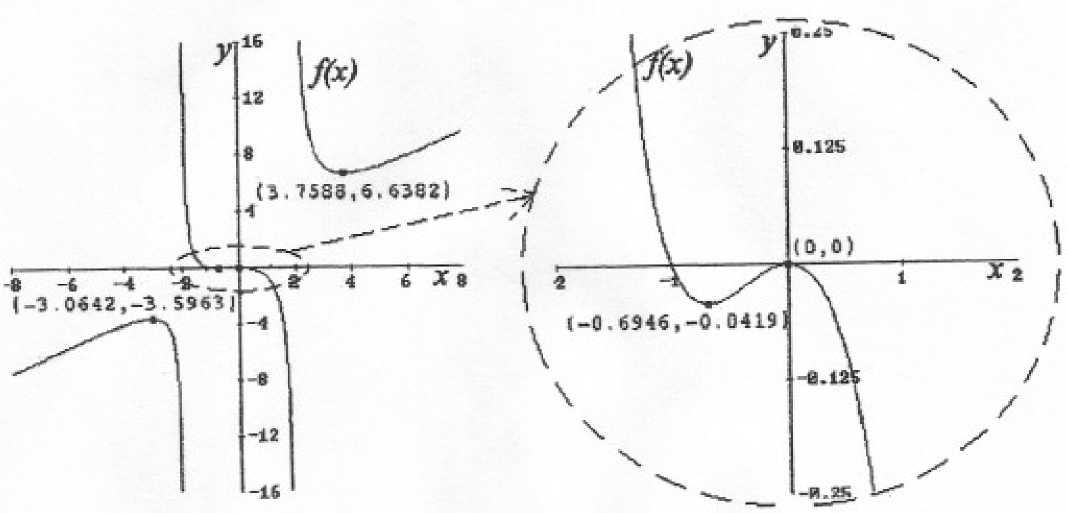
Problem 3.

(a) The graph has no zeros and vertical asymptotes at $x = -1$ and $x == 2$. The sign of y changes across both of these vertical asymptotes, which implies that there are odd powers of $(x+1)$ and $(x-2)$ in the denominator of the function. As there is a horizontal asymptote at $y = 0$, the degree of the numerator of the function is less than the degree of the denominator of the function. Hence the equation has the form $y = \frac{k}{(x+1)(x-2)}$, where k is a nonzero constant. As $y < 0$ for $|x| > 2$, $k < 0$. For simplicity, we choose $k = -1$. Therefore, $y = -\frac{1}{(x+1)(x-2)}$.

(b) The graph has simple zeros at $x = -2$ and $x = 0$ and a vertical asymptote at $x = -1$. As the sign of y does not change across the vertical asymptote, there is an even power of $(x+1)$ in the denominator of the function. As there is a horizontal asymptote at $y = 2$, the degrees of the numerator and denominator of the function are equal, and the lead the coefficient of the numerator is 2. Hence $y = \frac{2x(x+2)}{(x+1)^2}$.

(c) The graph has a simple zero at $x = -2$ and an even-ordered zero at $x = 0$. Thus, the numerator has factors of $(x+2)$ and x^2. There are vertical asymptotes at $x = -1$ and $x = 1$. The sign of y changes across the vertical asymptote $x = 1$ but not across $x = -1$. Thus, the denominator has factors of $(x+1)^2$ and $(x-1)$. As there is a horizontal asymptote at $y = 2$, the degrees of the numerator and denominator of the function are equal, and the lead the coefficient of the numerator is 2. Therefore, the equation has the form $y = \frac{2x^2(x+2)}{(x+1)^2(x-1)}$.

Problem 5.



Calculating the derivative of $f(x) = \frac{x^3+x^2}{x^2-4}$ via the quotient rule gives:

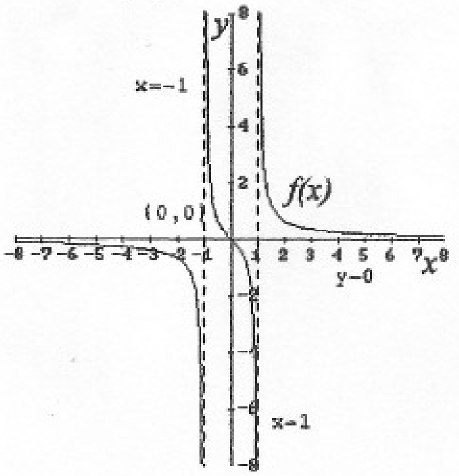
$$f'(x) = \frac{(3x^2+2x)(x^2-4)-(x^3+x^2)(2x)}{(x^2-4)^2} = \frac{(3x^4+2x^3-12x^2-8x)-(2x^4+2x^3)}{(x^2-4)^2} =$$

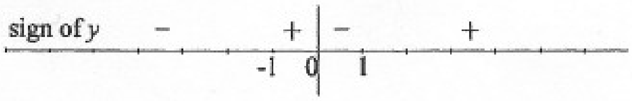
$$\frac{x^4-12x^2-8x}{(x^2-4)^2} = \frac{x(x^3-12x-8)}{(x^2-4)^2} = \frac{x^4-12x^2-8x}{(x^2-4)^2}.$$

Now, as $f'(0) = 0$, $x = 0$ is a critical point. A graphing calculator will verify that there is a local minimum at $x = 0$. Furthermore, a graphing calculator will find a local maximum at $x \approx -3.0642$, a local minimum at $x \approx -0.6946$, and a local minimum at $x \approx 3.7588$.

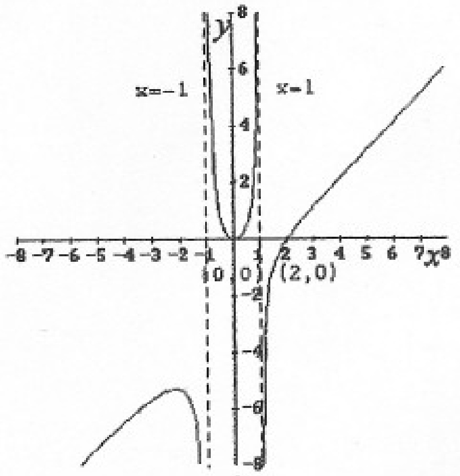
Problem 12.

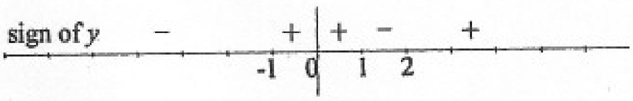
(i) The x-intercept is the origin; the y-intercept is the origin; the vertical asymptotes are $x = \pm 1$; the horizontal asymptote is the x-axis.



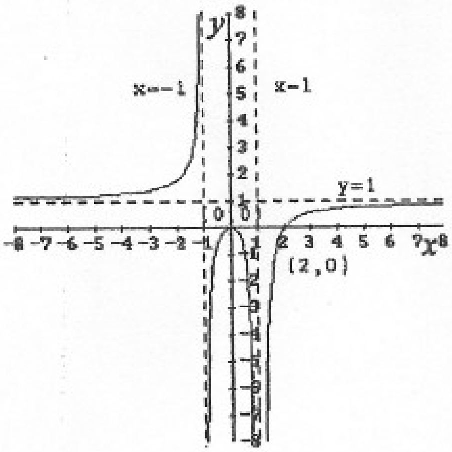


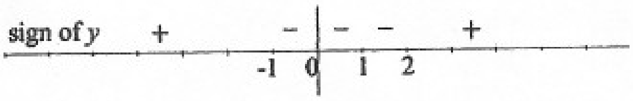
(ii),(iii) The x-intercepts are $x = 0, 2$; the y-intercept is the origin; the vertical asymptotes are $x = \pm 1$; there are no horizontal asymptotes.





(iv) The x-intercepts are $x = 0, 2$; the y-intercept is the origin; the vertical asymptotes are $x = \pm 1$; the horizontal asymptote is $y = 1$.





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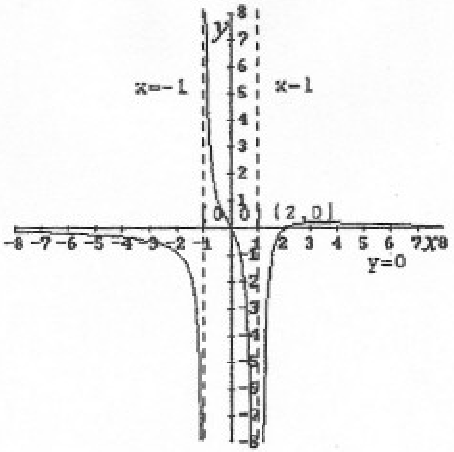
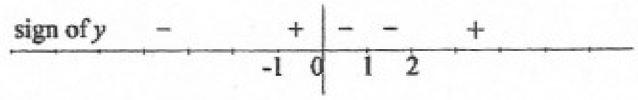
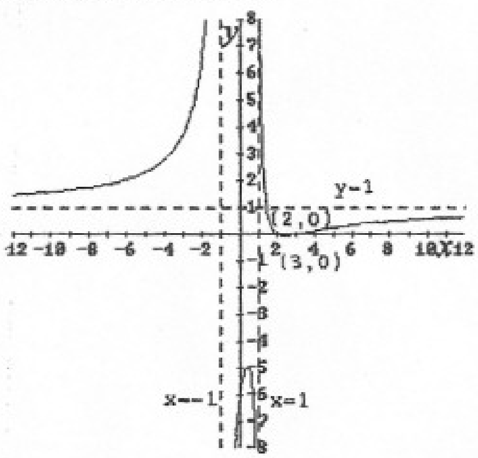
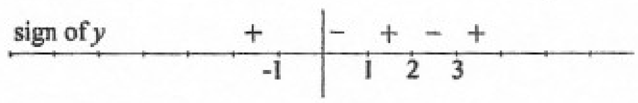
(v) The x-intercepts are $x = 0, 2$; the y-intercept is the origin; the vertical asymptotes are $x = \pm 1$; the horizontal asymptote is the $x-$axis.



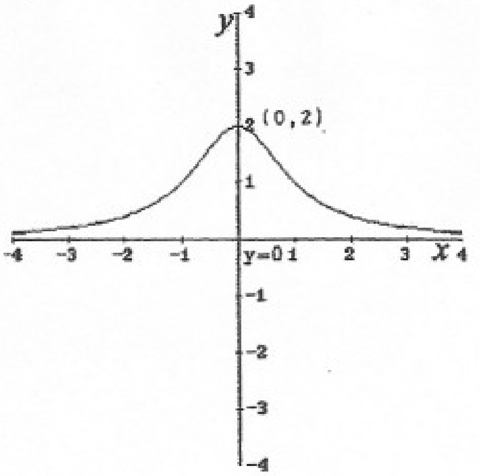


(vi) The x-intercepts are $x = 2, 3$; the y-intercept is $y = -6$; the vertical asymptotes are $x = \pm 1$; the horizontal asymptote is $y = 1$.





(vii) There are no x-intercepts; the y-intercept is $y = 2$; there are no vertical asymptotes; the horizontal asymptote is the x-axis. Note that $y < 0$ for all values of x.



(viii) The x-intercept is the origin; the y-intercept is the origin; there are no vertical asymptotes; the horizontal asymptote is $y = -1$. Note that $y < 0$ for all values of x.