

Simplify the following expressions without using a calculator.

1. $\frac{t}{\sqrt{t}}$, for $t > 0$

2. $\left(\frac{a^{\frac{3}{2}} a^{-\frac{1}{2}} b^2}{c^{-2}}\right)^5$, $c \neq 0$

3. $a^2\sqrt{a}$, $a \geq 0$

4. $\sqrt[3]{\frac{1}{a^9}}$, $a \neq 0$

Find all the solutions for the following equations without using a calculator. (If there are no solutions, make a brief argument about why there are none.)

1. $(-x)^{1/2} = 9$

2. $x^3 + 4x^2 - 5x = 0$

3. $\frac{1 - \frac{1}{x}}{1 - \frac{1}{x^2}} = 2$

Answer the following questions.

1. List all possible values of a and b for which the equation $ax^2 + bx + 1 = 0$ has

(a) No solutions

(b) One solution

(c) Two solutions

(d) More than two solutions

2. Find values (you don't have to list all values, you just have to find one example for each situation) for $a, b,$ and c for which the inequality $ax^2 + bx + c > 0$

(a) Has no solutions

(b) Is satisfied for every possible value of x

(c) Has infinitely many solutions, but not all values of x satisfy the inequality (include the solutions of your inequality)

(d) Do any of your answers change if the inequality is switched to $ax^2 + bx + c \geq 0$?

Reduce the following expressions to single fractions and simplify.

1.
$$\frac{\frac{a^3}{1+b}}{\frac{a^2}{1+2b+b^2}}$$

2.
$$\frac{3}{2\sqrt{x}} - \frac{1}{4(x+1)^2} + \frac{\sqrt{x}}{x+1}$$