Name:________________________________

Math Xa Final Examination—Tuesday, January 27, 2004

Please circle your section:

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Margaret Barusch (CA)	Connie Zong (CA)	Jennie Schiffman (CA)
MWF 11–12	MWF 12–1	MWF 11–12

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Evan Hepler-Smith (CA)	Maria van Wagenberg (CA)
MWF 10–11	MWF 10–11

Problem Number	Possible Points	Score
1	8	
2	6	
3	12	
4	6	
5	9	
6	10	
7	5	
8	5	
9	6	
10	5	
11	5	
12	9	
13	9	
14	6	
15	4	
Total	105	

Directions—Please Read Carefully! You have two hours to take this midterm. Make sure to use correct mathematical notation. Any answer in decimal form must be accurate to three decimal places, unless otherwise specified. To receive full credit on a problem, you will need to justify your answers carefully—unsubstantiated answers will receive little or no credit (except if the directions for that question specifically say no justification is necessary, such as in a True/False section). Please be sure to write neatly—illegible answers will receive little or no credit. If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of this on the problem page so that the grader knows where to find your answers. You may use a calculator on this exam, but no other aids are allowed. **Good Luck!!!**

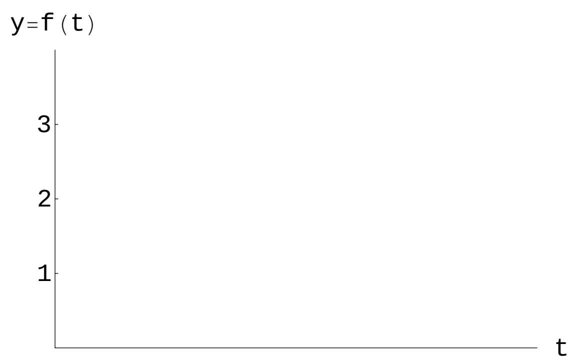
1. (8 points) A small furniture business signs a contract with customer to deliver up to 400 chairs. The exact number of chairs will be determined by the customer later. The price will be \$90 per chair up to 300 chairs, and above 300, the price will be reduced by \$0.25 per chair (on the entire order) for every additional chair over 300 ordered.

   (a) Write a function that expresses the revenue in terms of the number of chairs ordered.

   (b) What is the largest revenue that the company can make under this deal?

2. (6 points) Consider the vase shown below. Assume that the vase is filled with water at a constant rate (constant volume per unit time).

(a) Graph $y = f(t)$, the depth of the water, against time, t. Show on your graph the points at which concavity changes.



(b) At what depth is $y = f(t)$ growing most quickly? Most slowly?

3. (12 points) Evaluate each of the following expressions.

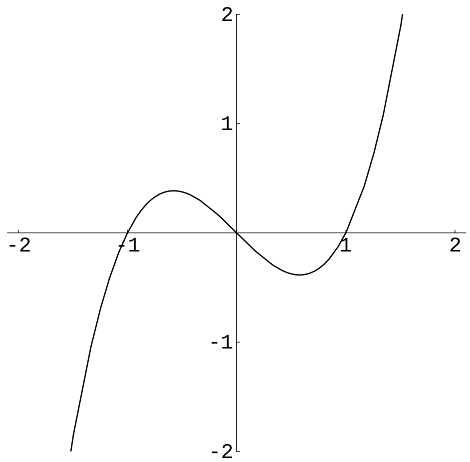
(a) Let $h(x) = f(x)e^{-x^2}$. If $f(2) = 1$ and $f'(2) = -2$, find $h'(2)$.

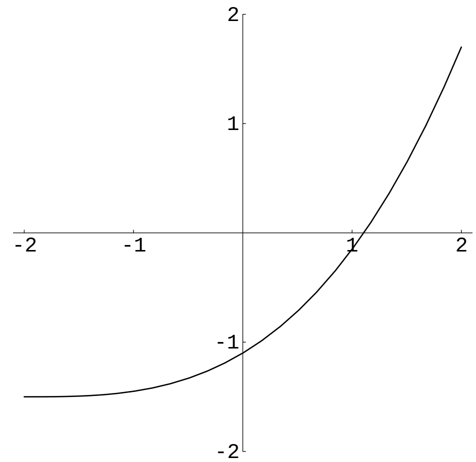
(b) Let $h(x) = f(x)/2^x$. If $f(2) = 1$ and $f'(2) = -2$, find $h'(2)$.

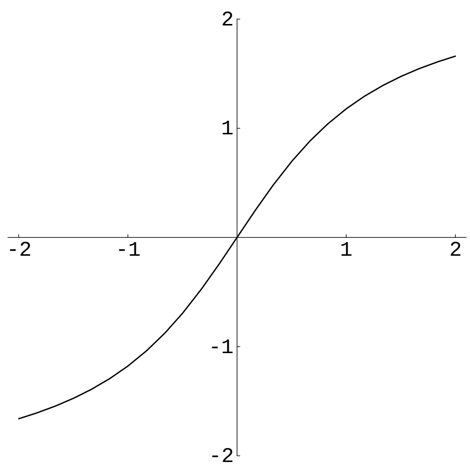
(c) Let $h(x) = f(2x^2 - 3x)e^x$. If $f(2) = 1$ and $f'(2) = -2$, find $h'(2)$.

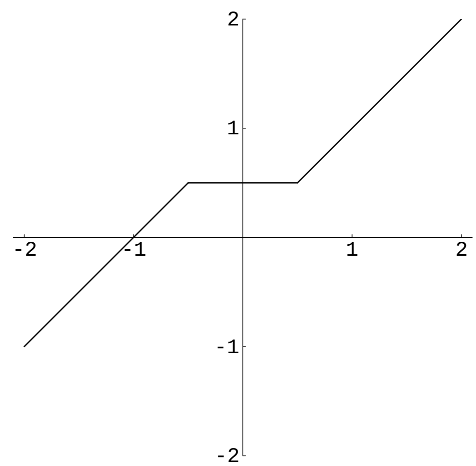
(d) Let $h(x) = \ln(f(x)^3)$. If $f(2) = 1$ and $f'(2) = -2$, find $h'(2)$.

4. (6 points) Which among the following graphs represents an *invertible* function? For each graph representing an invertible function, sketch the graph of the function's inverse on the same set of axes.









5. (9 points) Let f be a positive, increasing, and differentiable function such that $f(0) = 1$ and

$$g(x) = e^{2x} f(x^3).$$

(a) Find $g'(0)$.

(b) Find an equation of the tangent line to $g(x)$ at $x = 0$.

(c) Show that the function g is also an increasing, positive function.

6. (10 points) Let $f(x) = e^x(x^2 + x - 5)$.

(a) Find the x-values, if any, at which f has an absolute minimum.

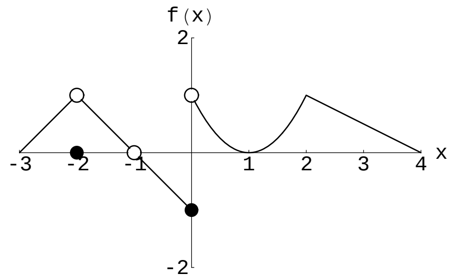
(b) Find the x-values, if any, at which f has an absolute maximum.

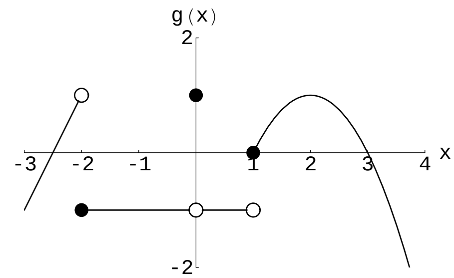
(c) Find the x-values, if any, at which f has a local minimum.

(d) Find the x-values, if any, at which f has a local maximum.

(e) Find the intervals, if any, on which f is concave up.

7. (5 points) Let f and g be the functions whose graphs are given below. Use the graphs to evaluate the limits. If the limit does not exist, say why.





(a) $\lim_{x \to 1} f(x) + g(x)$

(b) $\lim_{x \to -2^-} f(x) + g(x)$

(c) $(f + g)(-2)$

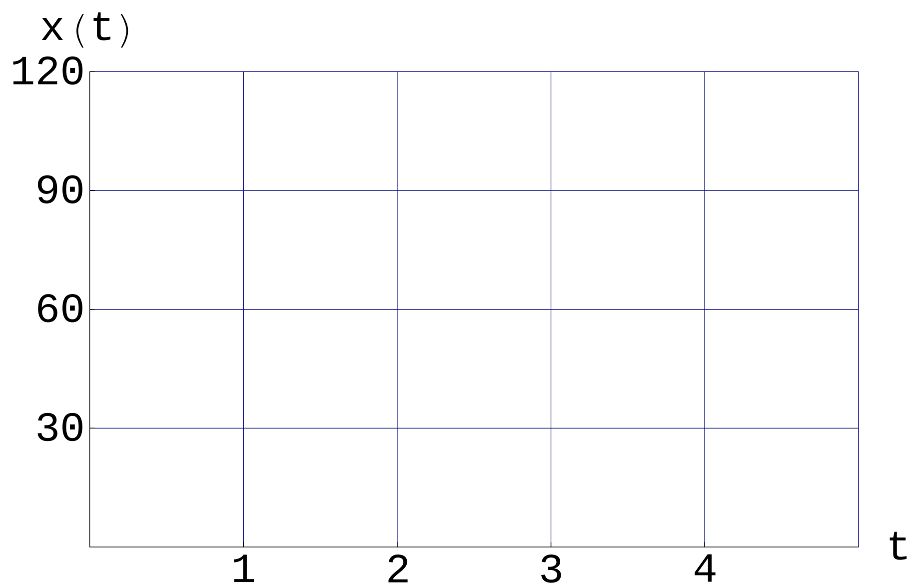
(d) $\lim_{x \to 2} f(x)g(x)$

(e) $\lim_{x \to 0} \dfrac{f(x)}{g(x)}$

8. (5 points) The function $x(t)$ describes a trip along a straight road that you made between two cities that are 120 miles apart:

> You left home and drove at 60 mi/hr for a half an hour. You realized that you had left your wallet at home and drove back at 60 mi/hr to get it. Then you drove at 60 mi/hr towards your destination for one hour when a herd of caribou crossed the road ahead of you. You stopped your car for a half an hour to wait for the herd to cross the road. After the herd crossed, you continued to your destination at 75 mi/hr.

Sketch the graph of the distance x from your starting point (in miles) as a function of time t elapsed (in hours).



9. (6 points) Use the *formal* definition of the derivative to show that $f'(1) = -1/16$, if

$$f(x) = \frac{1}{x+3}.$$

You must calculate a difference quotient and find a limit.

10. (5 points) Sketch the graph of a single function f that satisfies all of the following conditions.

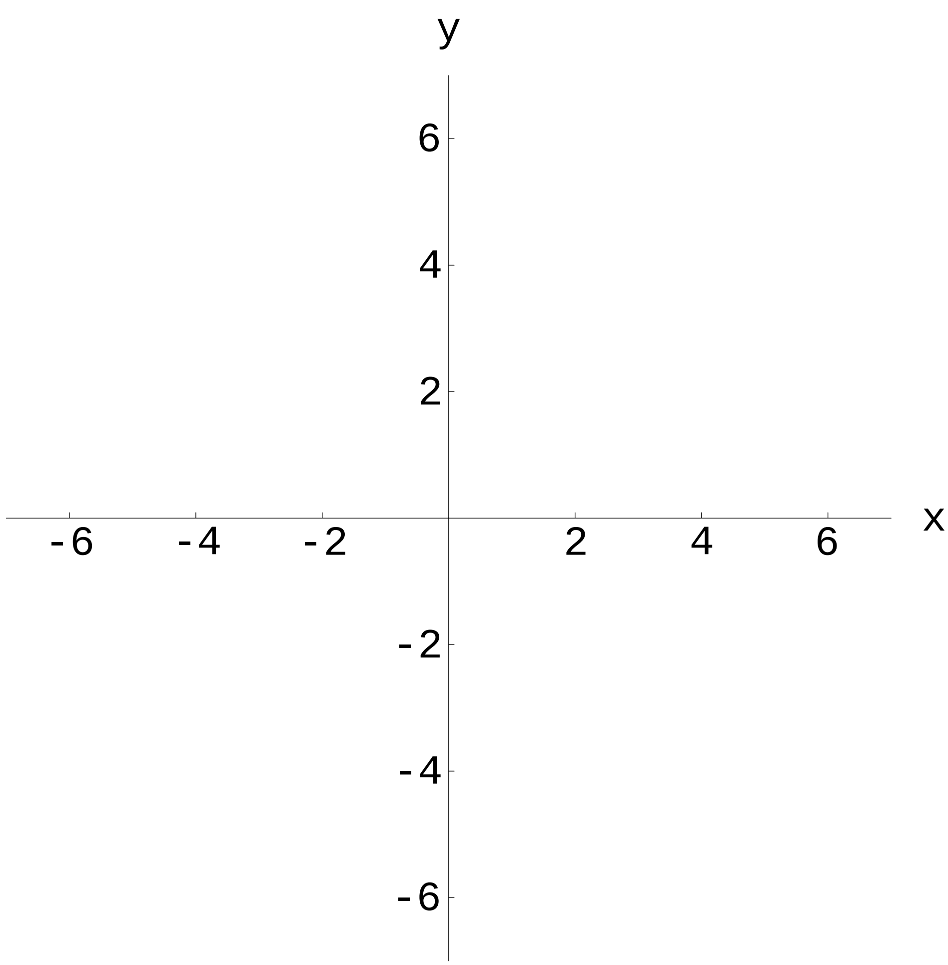
(a) $\lim_{x \to -2^-} f(x) = \infty$

(b) $\lim_{x \to -2^+} f(x) = -\infty$

(c) $\lim_{x \to \infty} f(x) = 3$

(d) $\lim_{x \to -\infty} f(x) = 3$

(e) $x = 1$ is a removable discontinuity of f



11. (5 points) Sketch the graph of a single function f that satisfies all of the following conditions.

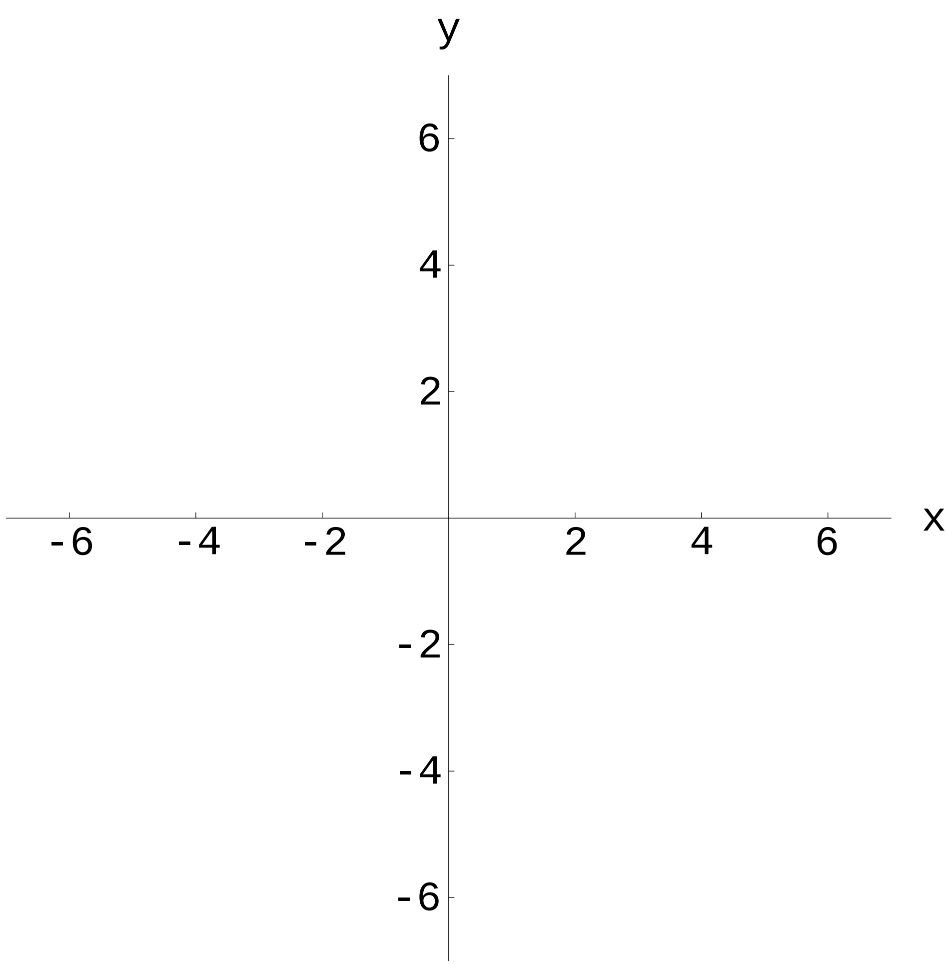
(a) $f'(x) > 0$ for all x in $(0, 4)$

(b) $f'(x) < 0$ for all x in $(4, \infty)$

(c) $f''(x) > 0$ for all x in $(-2, 0)$

(d) $f''(x) < 0$ for all x in $(-\infty, -2)$

(e) f is an even function



12. (9 points) An oil spill expands over the surface of a lake. Its area (in square meters) at t hours after the initial spill is given by $A(t) = 60000 \ln(t+1)$.

(a) Find the average rate of change in area during the first two hours of the spill.

(b) Find the instantaneous rate of change in area at two hours after the initial spill.

(c) If left unchecked, will the oil spill eventually cover the surface of the lake? Explain your answer.

13. (9 points) A certain bacteria culture grows exponentially. The bacteria count was 400 after two hours and 25,600 after six hours.

    (a) What was the initial population of the culture?

    (b) Find an expression for the population after t hours.

    (c) In what time period does the population double?

14. (6 points) When your morning coffee is poured, its temperature is 190°F. You sit down to enjoy your coffee in a nice 70°F room. After ten minutes the temperature of your coffee is 179°F and after ten minutes and six seconds the coffee has cooled to 177.9°F. Use linear approximation to estimate the temperature of your coffee after eleven minutes.

15. (4 points) A cubic polynomial is flipped over the horizontal axis, then shifted down by 3 units, and finally shifted to the right by 4 units. If the equation of the new polynomial is

$$y = -(x-1)^3 - 1,$$

what is the equation of the original polynomial?