Name: SOLUTIONS

Math Xa Midterm Examination I—Thursday, October 23, 2003

Please circle your section:

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MWF 11–12	MWF 12–1	MWF 11–12

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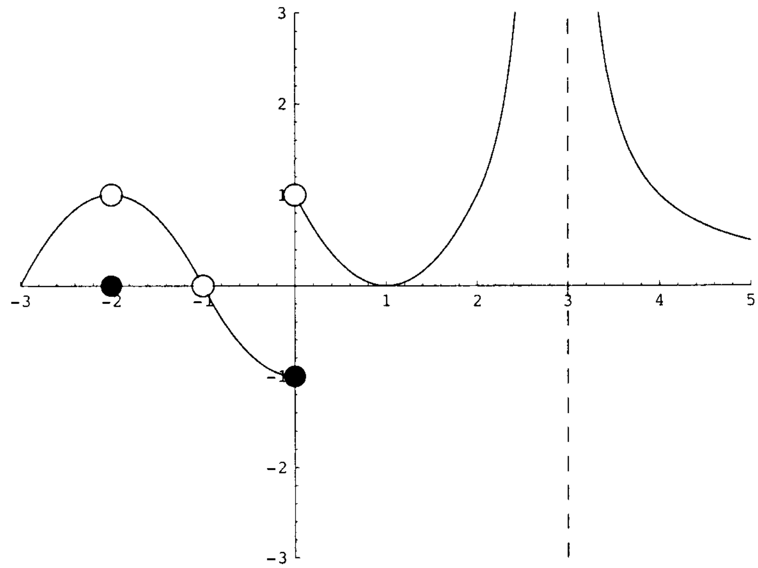
Problem Number	Possible Points	Score
1	10	
2	8	
3	10	
4	8	
5	5	
6	9	
7	10	
8	5	
9	9	
10	8	
11	8	
Total	90	

Directions—Please Read Carefully! You have two hours to take this midterm. Make sure to use correct mathematical notation. Any answer in decimal form must be accurate to three decimal places, unless otherwise specified. Pace yourself by keeping track of how many problems you have left to go and how much time remains. You do not have to answer the problems in any particular order, so move to another problem if you find you are stuck or spending too much time on a single problem. To receive full credit on a problem, you will need to justify your answers carefully—unsubstantiated answers will receive little or no credit (except if the directions for that question specifically say no justification is necessary, such as in the True/False section). Please be sure to write neatly—illegible answers will receive little or no credit. If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of this on the problem page so that the grader knows where to find your answers. You may use a calculator on this exam, but no other aids are allowed. **Good Luck!!!**

1. (10 points) Consider the graph of f given below.



(a) On what intervals is f positive?

$(-3,-2)$, $(-2,-1)$, $(0,1)$, $(1,3)$, $(3,5]$

(b) On what intervals is f decreasing?

$(-2,-1)$, $(-1,0]$, $(0,1]$, $(3,5]$

(c) On what intervals is f concave up?

$(-1,0]$, $(0,3)$, $(3,5]$

(d) List three x-values where f is not continuous.

$x = -2, -1, 0, 3$

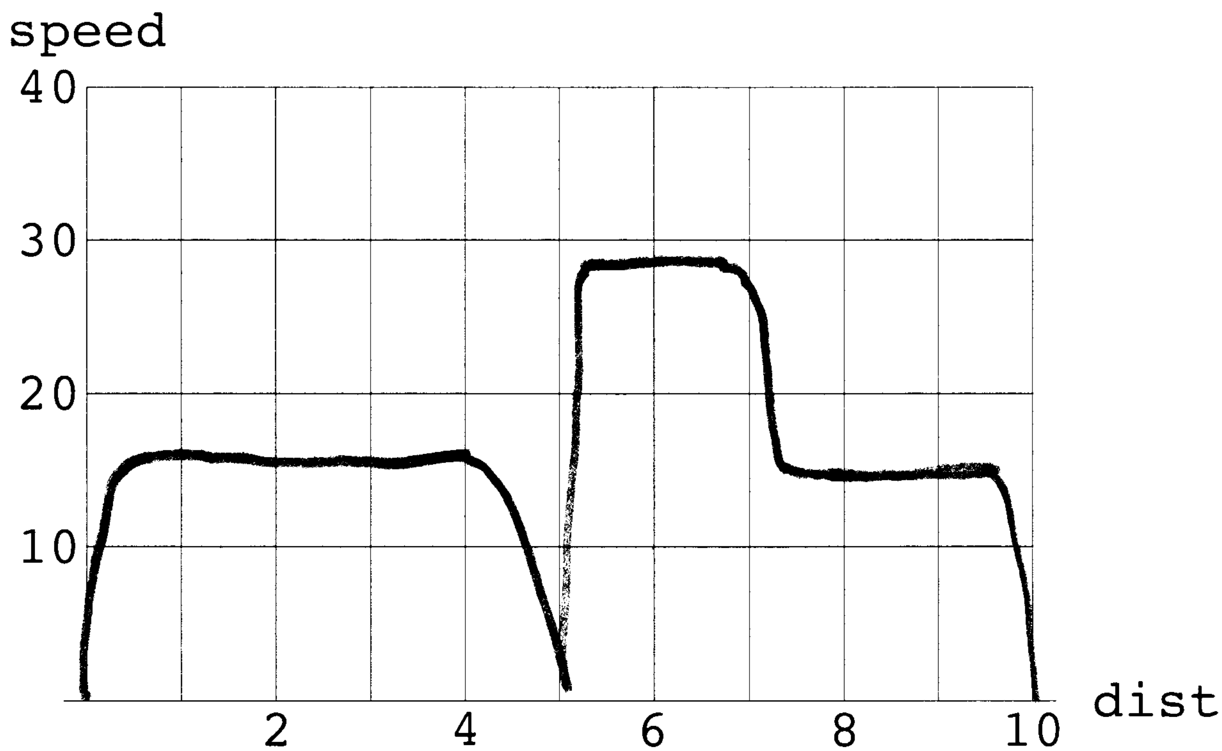
(e) At what x-value does f have a vertical asymptote?

$x = 3$

2. (8 points) Let $f(x) = x^2 - 3x$, and let $g(x)$ be defined by the following table. Fill in the missing entries in the table if possible. If an entry cannot be filled in, say why.

x	-1	0	2	3
$f(x)$	4	0	-2	0
$g(x)$	0	1	2	3
$(f/g)(x)$	CANNOT DIVIDE BY ZERO	0	-1	0
$f(g(x))$	0	-2	-2	0

3. (10 points) A Doctor rides his bicycle from his home to the clinic, a trip of about ten miles. He goes along at a constant speed until he reaches the big hill which starts at mile 4. He slows down as he goes up the hill, which peaks at mile 5. He is barely moving when he reaches the top and stops to rest for a bit. He then goes down the hill (fairly quickly thanks to the steep grade), reaching the bottom at mile 7. The last three miles to his office are flat. Sketch the graph of the function which shows the doctor's *speed* as a function of the *distance* from home.



4. (8 points) Let $f(x) = \dfrac{1}{x+1}$ and $g(x) = \sqrt{x-2}$.

(a) Find the domain of the function f.

$x \neq -1$

(b) Find the domain of the function g.

$x \geq 2$

(c) Find a formula for $(f \circ g)(x) = f(g(x))$.

$$f(g(x)) = f(\sqrt{x-2}) = \frac{1}{\sqrt{x-2}+1}$$

(d) Using your answer for part (c), find the domain of the function $(f \circ g)(x) = f(g(x))$.

$x \geq 2$

5. (5 points) If $f(x) = x^2 - 2x + 2$, find

$$\frac{f(a+h) - f(a)}{h}.$$

Simplify your answer.

$$\frac{f(a+h) - f(a)}{h}$$

$$= \frac{\left[(a+h)^2 - 2(a+h) + 2\right] - \left[a^2 - 2a + 2\right]}{h}$$

$$= \frac{\cancel{a^2} + 2ah + h^2 - \cancel{2a} - 2h + \cancel{2} - \cancel{a^2} + \cancel{2a} - \cancel{2}}{h}$$

$$= \frac{2ah + h^2 - 2h}{h}$$

$$= \frac{(2a + h - 2)h}{h}$$

$$= \boxed{2a + h - 2}$$

6. (9 points) Recall that an *odd function* is one that is symmetric about the origin. Equivalently, we can say that f is an odd function if it satisfies the relation $f(-x) = -f(x)$.

(a) Let g be an odd function. Some of the values of g are shown in the following table. Fill in the missing entries if possible. If not possible, say why.

x	-5	-4	-3	-2	-1	1	2	3	4	5
$g(x)$	6	5	1	3	-2	2	-3	-1	-5	-6
$g^{-1}(x)$	4	-7	2	-1	3	-3	1	-2	7	

$g(4) = -5 \Rightarrow g(-4) = -(-5) = 5$

$g(-2) = 3 \Rightarrow g^{-1}(3) = -2$

etc.

(b) Recall that an *even function* is one that is symmetric about the y-axis. Equivalently, we can say that f is an ~~odd~~ function if it satisfies the relation $f(-x) = f(x)$. Let $f(x)$ and $g(x)$ be odd functions. Show that their product,

EVEN

$$(fg)(x) = f(x)g(x),$$

must be an even function.

$(fg)(-x) = f(-x)\,g(-x)$

$= [-f(x)][-g(x)]$

$= f(x)\,g(x)$

$= (fg)(x)$

7. (10 points) Which of the following rules represents a function? For each rule that is not a function, explain why not.

(a) The rule that takes a year as input and gives back the age you were during that calendar year.

NOT A FUNCTION. YOU CAN BE TWO DIFFERENT AGES IN THE SAME CALENDAR YEAR.

(b) The rule that takes an age in years as input as input and gives back the calendar year in which you were that age.

NOT A FUNCTION. YOU CAN BE THE SAME AGE DURING TWO DIFFERENT CALENDAR YEARS.

(c) The rule that assigns to each person in your math class his or her age at 7 p.m. on October 23, 2003.

FUNCTION. YOU CAN ONLY BE ONE AGE AT A PARTICULAR POINT IN TIME.

(d) $f(x) = y$, where $y^4 = x$.

NOT A FUNCTION SINCE $(-1)^4 = (1)^4 = 1$

$f(1) = 1$ OR $f(1) = -1$.

(e) $f(x) = y$, where $y^3 = x$.

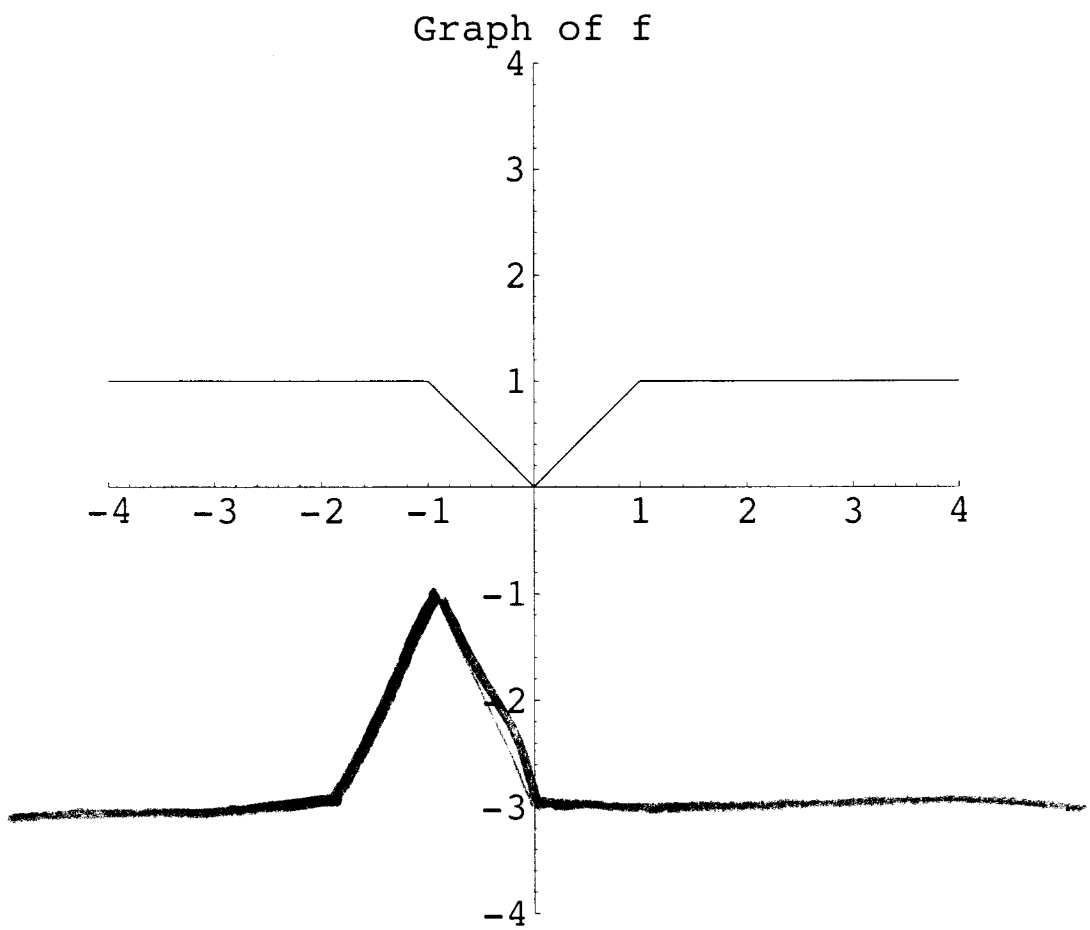
FUNCTION $y^3 = x \Rightarrow y = \sqrt[3]{x} = f(x)$

IS A FUNCTION.

8. (5 points) The graph of $y = f(x)$ is given below. Sketch the graph of

$$y = -2f(x+1) - 1$$

on the same axis.



9. (9 points)

(a) Find the equation of a line that is *parallel* to $y + 4x = 7$ passing through the point $(1, 5)$.

$$y = -4x + 7 \implies y - 5 = -4(x-1)$$

$$\implies y = -4(x-1) + 5$$

$$\implies \boxed{y = -4x + 9}$$

(b) Find the equation of a line that is *perpendicular* to $y + 4x = 7$ passing through the point $(1, 5)$.

$$y = -4x + 7 \implies y - 5 = \frac{1}{4}(x-1)$$

$$\implies y = \frac{1}{4}(x-1) + 5$$

$$\implies \boxed{y = \frac{1}{4}x + \frac{19}{4}}$$

(c) Owners of an inactive quarry in Australia have decided to resume production. They estimate that it will cost them \$1000 a month to maintain and insure their equipment and that monthly salaries will be \$3000. It costs \$80 to mine one ton of rocks. Write a function that expresses the total cost each month, c, as a function of r, the number of tons of rock mined per month.

$$c(r) = 1000 + 3000 + 80r$$

$$\boxed{c(r) = 4000 + 80r}$$

10. (8 points) Suppose that you are taking a long car trip and traveling west on Interstate 90, which runs from Boston to Seattle.

- Let f be the function that gives the number of miles traveled t hours into the trip, where $t = 0$ denotes the beginning of the trip. For example, $f(7)$ is the distance traveled seven hours into the trip.
- Let g be the function that gives the car's speed t ~~miles~~ hours into the trip, where $t = 0$ indicates the time at the beginning of the trip. For example, $g(7)$ is the speed of the car seven hours into the trip.

Suppose that the car passes a sign that reads "Entering Livingston, Montana," h hours into the trip.

Write the following expressions using functional notation wherever appropriate.

(a) The car's speed one hour before entering Livingston.

$g(h-1)$

(b) 10 miles an hour slower than the speed of the car entering Livingston.

$g(h)-10$

(c) The distance traveled five miles before entering Livingston.

$f(h)-5$

(d) The distance traveled five hours before entering Livingston.

$f(h-5)$

Interpret the following in words without using any references to symbols or variables.

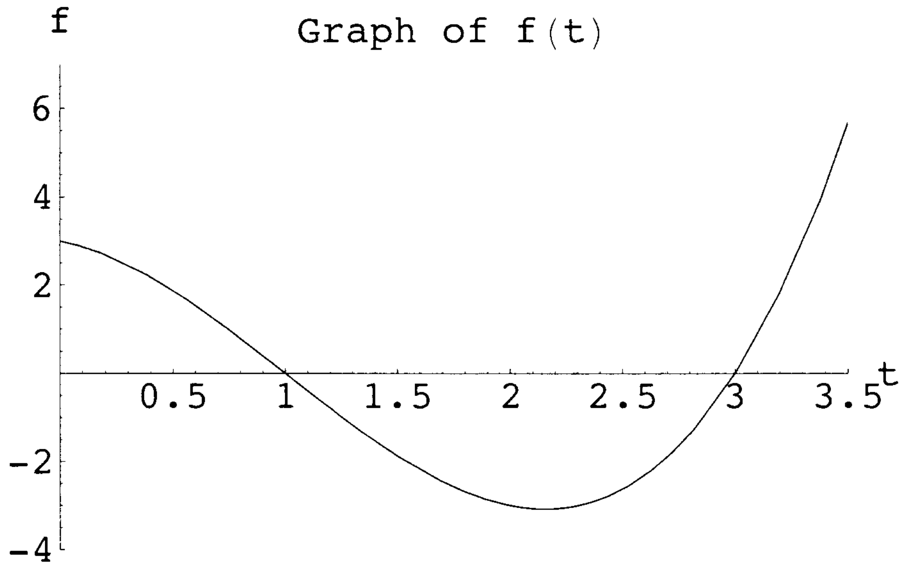
(e) $f(h+6)$ THE DISTANCE TRAVELED SIX HOURS AFTER ENTERING LIVINGSTON

(f) $f(h)-10$ TEN MILES BEFORE LIVINGSTON

(g) $g(h-8)$ THE SPEED 8 HOURS BEFORE ENTERING LIVINGSTON

(h) $\frac{1}{2}g(h)$ EXACTLY HALF THE SPEED YOU WERE TRAVELING WHEN YOU ENTERED LIVINGSTON.

11. (8 points) Consider the graph of $f(t)$, given below, which describes the motion of an object along the x-axis. All measurements are given in feet and seconds.



(a) If $f(t)$ is the *position* function of an object moving along the x-axis, where is the object at $t = 1$?

$\boxed{x = 0}$

(b) If $f(t)$ is the *position* function of an object moving along the x-axis, what is the average velocity of the object from $t = 0$ to $t = 1$?

$$\frac{f(1) - f(0)}{1 - 0} = \frac{0 - 3}{1} = \boxed{-3 \text{ FT/SEC}}$$

(c) If $f(t)$ is the *velocity* function of an object moving along the x-axis, which direction is the object moving at $t = 2$?

$\boxed{\text{TO THE LEFT}}$

(d) If $f(t)$ is the *velocity* function of an object moving along the x-axis, how many times does the object change direction between $t = 0$ and $t = 3.5$?

$\boxed{\text{TWICE}}$ AT $t = 1$ AND $t = 3$.