

Math S-Xab Summer 2004
Worksheet: Exponential Functions
July 14, 2004

1. A biologist observes a colony of bacteria growing in a petri dish. The following chart gives the number of bacteria $P(t)$ in the dish t hours after the experiment begins.

t	$P(t)$
0	10
1	20
2	40
3	80
4	160

Assuming this rate of growth continues, find a formula for $P(t)$.

2. A certain menacing biological culture (we'll call it the Blob) grows at a rate proportional to its size. When it arrived unnoticed one Wednesday noon in Chicago, it weighed just 1 g. By 4:00 p.m. rush hour, it weighed 4 g. The Blob has its "eye" on the Sears Tower, a tasty morsel weighing 3,000,000,000,000 g (i.e. 3×10^{12} g). As soon as it weighs 1000 times as much (i.e. 3×10^{15} g), The Blob intends to *eat* the Sears Tower. By what time must The Blob be stopped? Will Friday's rush-hour commuters be delayed?
3. The following questions deal with the effects of compound interest.
- (a) Suppose you invest \$1000 in a bank account paying 3% interest annually. How much will the account be worth in 10 years?
- (b) Suppose the interest is compounded monthly, that is, $\frac{3}{12}$ % interest is applied every month. How much will the account be worth in 10 years?
- (c) Suppose the interest is compounded daily, that is, $\frac{3}{365}$ % interest is applied every month. How much will the account be worth in 10 years?
4. Carbon-14, with a half-life of approximately 5730 years, can be used to date organic remains on earth. This isotope of carbon decays exponentially.
- (a) Let C_0 denote the amount of radioactive carbon in an organism when it is alive. Find a formula for $C(t)$, the amount of radioactive carbon in this organism t years after it has died.
- (b) In 1988, three teams of scientists found the shroud of Turin, which was reported to be the burial cloth of Jesus, contained 91% of the amount of carbon-14 contained in a freshly made cloth of the same material.¹ How old is the shroud of Turin according to these data?
5. Find the derivative of each of the following functions.
- (a) $y = xe^{10x}$
- (b) $f(t) = e^t + e^\pi + \pi^e$
- (c) $x(t) = \frac{t^3 - 2t + 4}{e^{3t} + 2}$
- (d) $y = \frac{e^x - e^{-x}}{2}$
- (e) $f(x) = (x^2 - 2x + 5)e^x$

¹The New York Times, October 18, 1988