Math S-Xab Summer 2004
Handout: The Limit Laws
July 12, 2004

Suppose that c is a constant, n is a positive integer, and the limits

$$\lim_{x\to a} f(x) \qquad \text{and} \qquad \lim_{x\to a} g(x)$$

exist. Then

1. $\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$ *(Sum Law)*

2. $\lim_{x\to a}(f(x)-g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$ *(Difference Law)*

3. $\lim_{x\to a}(cf(x)) = c\lim_{x\to a} f(x)$ *(Constant Multiple Law)*

4. $\lim_{x\to a}(f(x)g(x)) = \lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$ *(Product Law)*

5. $\lim_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x)\neq 0$ *(Quotient Law)*

6. $\lim_{x\to a}(f(x))^n = \left(\lim_{x\to a} f(x)\right)^n$ *(Power Law)*

7. $\lim_{x\to a} c = c$ *(Special Limit #1)*

8. $\lim_{x\to a} x = a$ *(Special Limit #2)*

9. $\lim_{x\to a} x^n = a^n$ *(Power of x Law)*

10. $\lim_{x\to a}\sqrt[n]{x} = \sqrt[n]{a}$, where if n is even, then we assume $a>0$ *(Root of x Law)*

11. $\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$, where if n is even, then we assume $\lim_{x\to a} f(x)>0$ *(Root Law)*

Inequality Law: If $f(x)\le g(x)$ when x is near a (except possibly at a) and $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist, then

$$\lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$$

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The Squeeze (or Sandwich) Theorem: If $f(x)\le g(x)\le h(x)$ when x is near a (except possibly at a) and

$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L,$$

then

$$\lim_{x\to a} g(x) = L.$$