Math S-Xab
Worksheet—Differentiating Exponential and Logarithmic Functions

Summer 2004

1. Argue that

$$\frac{d}{dx}\ln(x+a) = \frac{1}{x+a},$$

   for any constant a.

2. Find the derivative of each of the following functions.

   (a) $y = x^e + \pi^x + e^\pi$

   (b) $y = x\log_3 x$

   (c) $y = \frac{1}{2}\ln x$

   (d) $y = \ln\sqrt{x}$

   (e) $y = \ln(x\sqrt{x})$

   (f) $y = \ln\left(\frac{x^2\sqrt{x}}{(x-2)(x+5)}\right)$

3. Suppose that $f'(x) = x + 5 \cdot 2^x$. Find the equation of a line tangent to $f(x)$ at the point $(0,3)$. Does this line lie above the graph of f or below the graph of f? How can you tell?

4. In early summer the fly population of Maine grows exponentially. The population at any time t (measured in days) can be given by $P = P_0 e^{kt}$ for some constant k. Suppose at some date, which we will designate as $t = 0$, there are 200 flies. Thirty days later, there are 900 flies.

   (a) Find P_0 and k.

   (b) The mosquito population is also growing exponentially. At time $t = 0$ there are 100 mosquitos, and the mosquito population doubles every 10 days. Write a function $M(t)$ that gives the number of mosquitos at time t.

   (c) When will the mosquito and fly populations be equal?

   (d) Find $P'(t)$.

   (e) Find $M'(t)$.

   (f) Find the rate at which each population is growing when the two populations are equal. Which population is growing more rapidly?