

Math S-Xab Summer 2004
Worksheet: Optimization
July 16, 2004

Optimization Problems Strategy

1. Understand the situation. Drawing a picture often helps.
2. Identify the variables—those quantities that change.
3. Identify the variable you wish to maximize or minimize. Express this variable in terms of the other variables.
4. You will probably have too many variables. Use the restrictions given in the problem to express the variable you wish to optimize as a function of only one variable.
5. Use the problem description to find the domain of this function.
6. Determine the maximum or minimum value of the function using techniques from §10.1 and §10.2.
7. Make sure to answer the precise question that was asked.

Examples

1. Farmer Brown has 120 meters of fencing with which to fence off a rectangular pen adjacent to the side of her barn. What dimensions of the pen yield the largest area?
2. A box with a square base holding 8000 cubic centimeters is to be manufactured. Find the dimensions that will minimize the surface area of the box.
3. Find the point on the line $y = 2x + 5$ that is closest to the origin.
4. A cylindrical can is to be made to hold one liter of oil. Find the dimensions that will minimize the cost of the metal used to manufacture that can.
5. Find two nonnegative numbers that add up to 66 such that their product is as large as possible.
6. You have just invented a new peanut butter guacamole dip, and you stand in front of the Science Center to sell your product by the jar. Somehow, a rumor gets started (not traceable to you) that your dip cures acne, baldness, and acts as a sedative for younger siblings. Sales take off.
At a price of \$1.00 per jar, you can sell 500 jars a day. For every nickel that you increase the price, you sell two fewer jars. Assuming that your fixed costs are \$200 per day (protection money), and the cost per jar is \$0.50, determine the price for which you should sell your dip in order to maximize profit.
7. A man launches his boat from point A on the bank of a straight river 3 kilometers wide. He wants to reach point B , 8 kilometers downstream on the opposite bank, as quickly as possible.
He could row his boat directly across the river to point C and then run to B , or he could row directly to B , or he could row to some point D between C and B and then run to B .
If he can row at 6 kilometers per hour and run at 8 kilometers per hour, where should he land to reach B as soon as possible? (Assume that the speed of the water is negligible compared to the speed at which the man rows.)