

Math S-Xab Summer 2004
Worksheet: Polynomials
July 19, 2004

- For each of the following functions, graph the function and its derivative on a graphing calculator. Then determine the number of critical points of the function.
 - $f(x) = 2x^3 + 6x + 6$
 - $g(x) = 2x^3 + 6$
 - $h(x) = 2x^3 - 9x^2 + 6$
- How many critical points can a generic cubic function $f(x) = ax^3 + bx^2 + cx + d$ have? Justify your answer.
- How many local extrema can a cubic function have? Justify your answer.
- How many inflection points can a cubic function have? Justify your answer.
- If $f(x) = ax^3 + bx^2 + cx + d$, what are $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$?
- How many x -intercepts can a cubic function have? Justify your answer.
- Suppose that f is a polynomial of degree n .
 - What is the maximum number of critical points f can have? Justify your answer.
 - What is the maximum number of turning points f can have? Justify your answer.
 - What is the maximum number of inflection points f can have? Justify your answer.
- If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, what are $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$? (Your answer should have several cases, depending on the values of a_n and n .)
- Sketch the graph of a cubic function f that ...
 - ... has roots at $x = -5$, $x = 2$, and $x = 6$.
 - ... has roots at $x = -1$ and $x = 1$ only and satisfies $f(0) = 3$.
 - ... has a roots at $x = 3$ only and satisfies $\lim_{x \rightarrow \infty} f(x) = -\infty$.
 - ... has a local minimum at $x = -1$ and a local maximum at $x = 3$.
 - ... has an inflection point at $x = 3$.
- Find an algebraic formula for each of the cubic functions described in Question 9.
- Find a formula for a fifth degree polynomial f with zeros of multiplicity two at $x = 0$ and $x = 7$ and a zero of multiplicity one at $x = -2$ such that $\lim_{x \rightarrow \infty} f(x) = \infty$.
- Find a formula for each of the polynomials whose graphs are given below.

