Answers to Practice Problems for the Gateway

1. Rewrite: $y = \frac{\ln 1 - 2\ln x}{e^{2x}} = -2\ln x\, e^{-2x}$

$y' = -2\left[\frac{1}{x}e^{-2x} + \ln x\,(-2)e^{-2x}\right]$

$y' = \frac{-2}{xe^{2x}} + \frac{4\ln x}{e^{2x}}$

2. Rewrite: $y = \pi^2\left(e^{7x+2}\right)^{-1/2} = e^{-\frac{1}{2}(7x+2)}$

$y' = \pi^2\, e^{-\frac{1}{2}(7x+2)} \cdot \left(-\frac{7}{2}\right)$

3. $x^2y = \frac{3e^{xy}}{\sqrt{3}} \iff x^2y = \frac{3}{\sqrt{3}}e^{xy}$

$x^2y = \sqrt{3}\,e^{xy}$ take logs

$\ln(x^2y) = \ln\sqrt{3}e^{xy}$

$2\ln x + \ln y = \ln\sqrt{3} + xy$

$\frac{2}{x} + \frac{1}{y}\frac{dy}{dx} = x\frac{dy}{dx} + y$

$\frac{dy}{dx}\left[\frac{1}{y} - x\right] = y - \frac{2}{x}$

$\frac{dy}{dx} = \frac{y - \frac{2}{x}}{\frac{1}{y} - x}$ or $\frac{xy^2 - 2y}{x - x^2y}$

$x^2\frac{dy}{dx} + 2xy = \frac{3}{\sqrt{3}}e^{xy}\left(x\frac{dy}{dx} + y\right)$

$x^2\frac{dy}{dx} + 2xy = \frac{3}{\sqrt{3}}e^{xy}x\frac{dy}{dx} + \frac{3}{\sqrt{3}}ye^{xy}$

$x^2\frac{dy}{dx} - \frac{3}{\sqrt{3}}e^{xy}x\frac{dy}{dx} = \frac{3}{\sqrt{3}}ye^{xy} - 2xy$

$\frac{dy}{dx} = \frac{\frac{3}{\sqrt{3}}ye^{xy} - 2xy}{x^2 - \frac{3}{\sqrt{3}}e^{xy}x}$

$\frac{3}{\sqrt{3}} \cdot \frac{3}{3^{1/2}} = 3^{1-\frac{1}{2}} = 3^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{\sqrt{3}ye^{xy} - 2xy}{x^2 - \sqrt{3}e^{xy}x}$

∴ Are these answers the same? Yes. $e^{xy} = \sqrt{3}\,x^2y$: simplify & you'll get what was on the

4. $y = \frac{x}{\ln(x+y)}$

$y\ln(x+y) = x$

$\frac{dy}{dx}\ln(x+y) + y\frac{1}{x+y}\left(1 + \frac{dy}{dx}\right) = 1$

$\frac{dy}{dx}\ln(x+y) + \frac{y}{x+y}\frac{dy}{dx} = 1 - \frac{y}{x+y}$

$\frac{dy}{dx} = \frac{1 - \frac{y}{x+y}}{\ln(x+y) + \frac{y}{x+y}}$ this can be simplified $\frac{dy}{dx} = \frac{x+y-y}{(x+y)\ln(x+y) + y}$ or $\frac{dy}{dx} = \frac{x}{(x+y)\ln(x+y)}$

5. $y = \frac{\pi}{\ln(x^2+4)^{1/2}}$ Rewrite: $y = \frac{\pi}{\frac{1}{2}\ln(x^2+4)} = \frac{2\pi}{\ln(x^2+4)} = 2\pi\left[\ln(x^2+4)\right]^{-1}$

$y' = 2\pi(-1)\left[\ln(x^2+4)\right]^{-2}\frac{1}{x^2+4}\cdot 2x$

$y' = \frac{-4\pi x}{\left[\ln(x^2+4)\right]^2\cdot(x^2+4)}$

6. $y' = \sqrt{\pi}\left(1 + \frac{0.1}{12}\right)^{12x}\ln\left(1 + \frac{0.1}{12}\right)\cdot 12$ or $\sqrt{\pi}\left[\left(1 + \frac{0.1}{12}\right)^{12}\right]^x \ln\left(1 + \frac{0.1}{12}\right)^{12}$

7. $y = 6x\,e^{\sqrt{x^2+3x}}$

$y' = 6\left[e^{\sqrt{x^2+3x}} + xe^{\sqrt{x^2+3x}}\cdot\frac{1}{2\sqrt{x^2+3x}}\cdot(2x+3)\right] = 6e^{\sqrt{x^2+3x}}\left(1 + \frac{2x+3}{\sqrt{x^2+3x}\cdot 2}\right)$