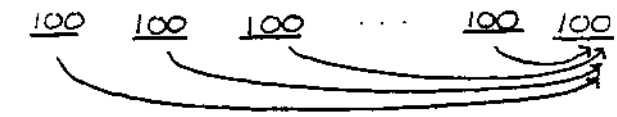
Solutions to Lab #4

1. 

let t = # of months a deposit is in the bank.

Each deposit grows according to $M(t) = 100(1.005)^t$

$100 + 100(1.005) + 100(1.005)^2 + \cdots + 100(1.005)^{35}$

↑ last payment; ↑ what the 2nd-to-last payment grew to; ↑ what the 3rd-from-last payment grew to; ↑ what the 1st payment grew to.

$100 + 100(1.005) + 100(1.005)^2 + \cdots + 100(1.005)^{35}$ is a geometric sum. with $r = (1.0$

$S = 100 + 100(1.005) + \cdots + 100(1.005)^{35}$

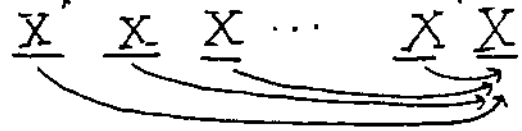
$1.005\,S = 100(1.005) + 100(1.005)^2 + \cdots + 100(1.005)^{36}$

$-.005\,S = 100 - 100(1.005)^{36}$

$S = \frac{100}{-.005}\left(1 - (1.005)^{36}\right)$

He will have saved $ 3933.61. It's hard to buy a baby grand with this.

2. Let X equal the amount of money Nick puts away each month.



let t = # of months a deposit is in the bank

Each deposit grows according to $M(t) = X\left(1 + \frac{.04}{12}\right)^t = X\left(1 + \frac{.01}{3}\right)^t$

Right after the 30th payment he has

(*) $X + X\left(1 + \frac{.01}{3}\right) + X\left(1 + \frac{.01}{3}\right)^2 + \cdots + X\left(1 + \frac{.01}{3}\right)^{29}$

↑ last payment

(*) gives the amount of money in his account right after the 30th payment.

We want this to be $ 3,000.

Put * in closed form:

$S = X + (X)\left(1 + \frac{.01}{3}\right) + \cdots + X\left(1 + \frac{.01}{3}\right)^{29}$

$\left(1 + \frac{.01}{3}\right)S = X\left(1 + \frac{.01}{3}\right) + \cdots + X\left(1 + \frac{.01}{3}\right)^{30}$

$S\left[1 - \left(1 + \frac{.01}{3}\right)\right] = X - X\left(1 + \frac{.01}{3}\right)^{30}$

$S = X\left[\frac{1 - \left(1 + \frac{.01}{3}\right)^{30}}{-\frac{.01}{3}}\right]$

So $X\left[\frac{3}{-.01}\left[1 - \left(1 + \frac{.01}{3}\right)^{30}\right]\right] = 3000 \Rightarrow X = \frac{3000}{\frac{3}{-.01}\left[1 - \left(1 + \frac{.01}{3}\right)^{30}\right]}$