$X \approx \$95.25$

This make sense - the answer is a bit less than $\frac{\$3000}{30} = \$100$, but not much less.

3. Borrow \$18,000. The interest grows according to $M(t) = M_0(1.08)^t$

Let P = annual payments

$18{,}000 \underbrace{\qquad\qquad}_{4\text{ yrs}} P \quad P \quad P \quad P \quad P \quad P$

Take One:

Yr	Balance Owed
0	$18{,}000$
4	$18{,}000(1.08)^4 - P$
5	$18{,}000(1.08)^5 - P(1.08) - P$
6	$18{,}000(1.08)^6 - P(1.08)^2 - P(1.08) - P$
7	$18{,}000(1.08)^7 - P(1.08)^3 - P(1.08)^2 - P(1.08) - P$
8	$18{,}000(1.08)^8 - P(1.08)^4 - P(1.08)^3 - P(1.08)^2 - P(1.08) - P$
9	$18{,}000(1.08)^9 - P(1.08)^5 - P(1.08)^4 - P(1.08)^3 - P(1.08)^2 - P(1.08) - P$

After 6 payments are made the debt is zero. Set the last line equal to zero. Equivalently

$$18{,}000(1.08)^9 - \left[P + P(1.08) + P(1.08)^2 + \cdots + P(1.08)^5\right] = 0$$

$$(*) \qquad 18{,}000(1.08)^9 = \underbrace{P + P(1.08) + \cdots + P(1.08)^5}$$

put in closed form $\frac{P - P(1.08)^6}{1 - 1.08}$

$$18{,}000(1.08)^9 = \frac{P - P(1.08)^6}{-.08}$$

$$18{,}000(1.08)^9 = P\left[\frac{1 - (1.08)^6}{-.08}\right]$$

$$P \approx \$4904.91$$

Take Two: $\$18{,}000 \underbrace{\qquad\qquad}_{4\text{ yrs.}} P \quad P \quad P \quad P \quad P \quad P$

The sum of the present values shou
be \$18,000

$$\frac{P}{(1.08)^4} + \frac{P}{(1.08)^5} + \frac{P}{(1.08)^6} + \frac{P}{(1.08)^7} + \frac{P}{(1.08)^8} + \frac{P}{(1.08)^9} = 18{,}000$$

[Notice - this i
eq'n (*) divi
by (1.08)

$$P\underbrace{\left[\frac{1}{(1.08)^4} + \frac{1}{(1.08)^5} + \cdots + \frac{1}{(1.08)^9}\right]} = 18{,}000$$

Put in closed form:

$$S = \frac{1}{(1.08)^4} + \frac{1}{(1.08)^5} + \cdots + \frac{1}{(1.08)^9}$$

$$-\quad \frac{1}{1.08} S = \frac{1}{(1.08)^5} + \cdots + \frac{1}{(1.08)^{10}}$$

$$S\left(1 - \frac{1}{1.08}\right) = \frac{1}{(1.08)^4} - \frac{1}{(1.08)^{10}}$$