

Mathematics Xb
Second Examination
April 13, 1995

1. (11 points) Find the derivatives of the following functions:

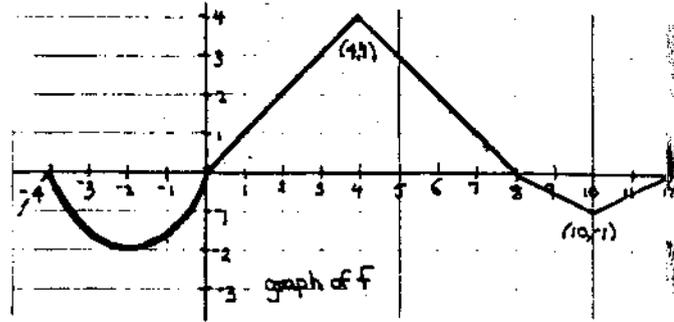
a) $f(x) = \frac{x^3 \sin(5x)}{3}$

b) $f(x) = \tan^3(\sqrt{x})$

c) $f(x) = 2 \arctan(x^5)$

2. (17 points) Consider the function f graphed below. (The graph consists of a semicircle and straight lines.)

Let $F(x) = \int_0^x f(t) dt$. Please answer the following questions. Be sure your answers are consistent.



a) Where on the interval $[-4, 12]$ is $F(x)$ increasing?

b) Where on the interval $[-4, 12]$ is $F(x)$ decreasing?

c) Where on the interval $[-4, 12]$ is $F(x)$ concave up?

d) Where on the interval $[-4, 12]$ is $\frac{dF}{dx} = 0$?

e) What is $F(-2)$?

f) Where on the interval $[-4, 12]$ does $F(x)$ attain its absolute maximum? What is its maximum value?

g) Where on the interval $[-4, 12]$ does $F(x)$ attain its absolute minimum? What is its minimum value?

h) If f is the rate of change of water in a reservoir at time t in gallons per hour, t measured in hours, what is the net change in water in the reservoir over the interval from $t = 4$ to $t = 12$?

3. (15 points) For each of the following, first determine whether your answer should be positive, negative, or zero, and then evaluate the integral.

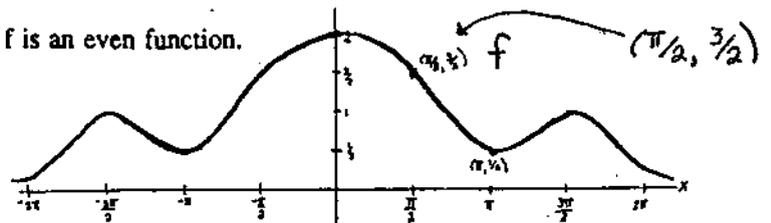
a) $\int_1^2 \left(\frac{1}{t^2} + \frac{1}{t} \right) dt$

b i) $\int_0^{\pi/2} \cos(x) dx$

b ii) $\int_{-\pi}^{2\pi} |\cos(x)| dx$

c) $\int_{-1}^0 \frac{2}{1+t^2} dt$

4. (16 points) The graph of $f(x)$ is provided below. f is an even function.



a) Put the following in ascending order. Put '<' or '=' signs between the integrals as is appropriate.

A = $\int_{-\pi}^{\pi} f(x) dx$

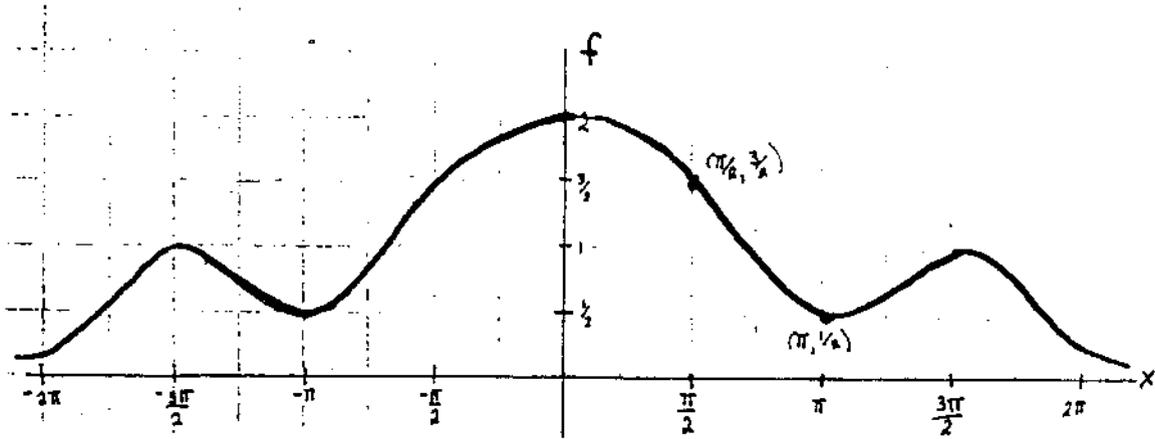
B = $\int_0^{\pi} f(x) dx$

C = $\int_0^{2\pi} f(x) dx$

D = $\int_{-\pi}^0 f(x) dx$

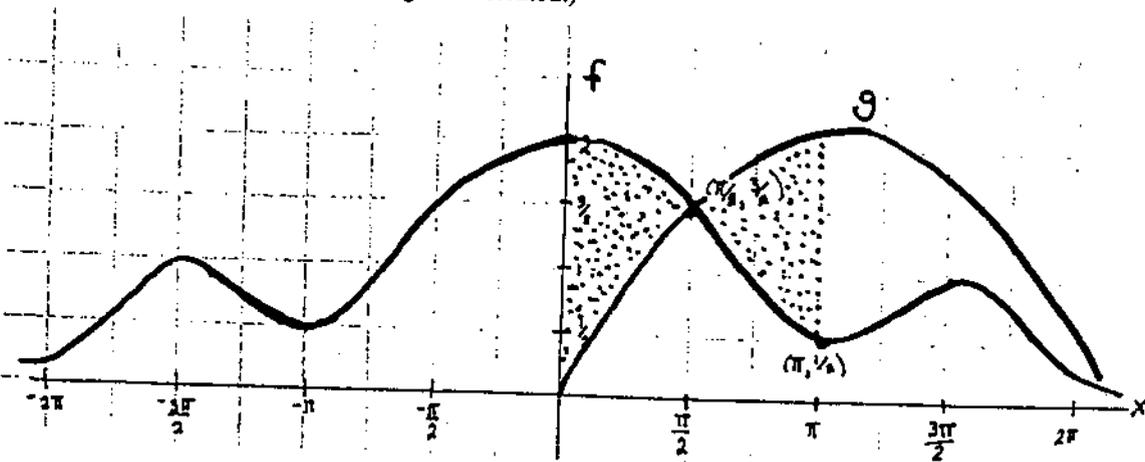
E = $\int_{\pi}^{2\pi} f(x) dx$

- b) Insert the integral $F = \int_{-2}^0 f(x) \sin(x) dx$ into the ordering done in part a)

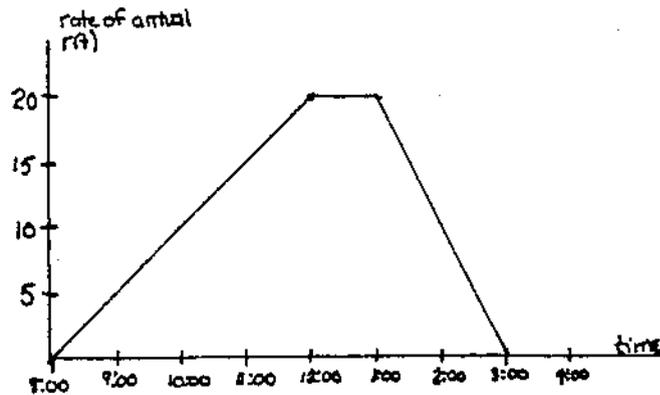


- c) Find upper and lower bounds for the integral $\int_0^{\pi} f(x) dx$ by calculating left and right hand Riemann Sums with two subdivisions (i.e. partition the interval $[0, \pi]$ into two equal pieces).

- d) Set up a definite integral or sum of definite integrals that gives the shaded area below (the area between the graphs of $f(x)$ and $g(x)$ in the region indicated.)

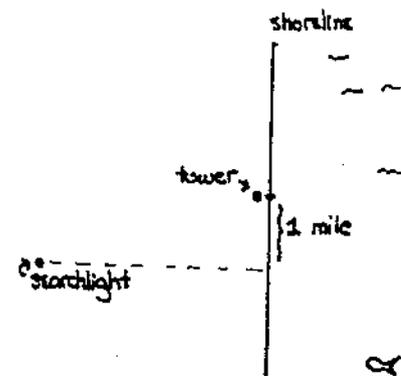


6. (11 points) A hospital clinic can admit patients at a rate of 10 patients per hour. The clinic opens at 8:00am and patients are admitted in the order in which they come. The rate at which patients are arriving at the clinic is given below.



- If you arrive at the clinic before 10:00am, will you have to wait on line? Explain.
- At what time is the line to be admitted the longest? At this time, how many people are on line? Explain your reasoning.
- What is the longest amount of time a patient must wait before being admitted?
- At what time in the afternoon will the line for the clinic disappear?
- Is the line increasing or decreasing at noon? At what rate?

7. (9 points) A rotating searchlight is located 3 miles from a straight shoreline. The searchlight passes by a tower located on the shoreline 1 mile from the point on the shore closest to the searchlight. The light passes the tower at a rate of 20 miles/minute. At what speed is the searchlight rotating? First give your answer in radians/minute and then convert to revolutions/minute.



8. (7 points) The first draft of a long paper is being written. Inspiration and clarity of thought is building and the writer puts words to paper at an ever increasing rate. Assume that since this is a first draft, the writer never erases any thing she has written. At 7:00 pm 100 words are on the paper. Below is a list of her writing rate, in words/hour. Note that the time intervals given are *not* equally spaced.

<u>TIME</u>	<u>WRITING RATE</u>
7:00 pm	50 words/hr
9:00 pm	60 words/hr
11:00 pm	100 words/hr
11:30 pm	120 words/hr
midnight	140 words/hr

Find good upper bound for the number of words on the paper at midnight. Find a good lower bound for the number of words on the paper at midnight. (You must be certain that she writes more than the lower bound and less than the upper bound that you give.)