ICE - Symbolic Solutions of DEs

Mathematical modeling is the process of creating a mathematical representation of a situation in order to develop a better understanding of how that situation works. In this ICE, you will build a model to describe what happens when people imbibe alcoholic beverages. Please note that this is a very simple model that ignores a lot of information on how the body absorbs, processes and eliminates alcohol.

Please do not use the results of this ICE as a guide for your own drinking behavior. People under the legal age should not consume alcoholic beverages without a special dispensation. If you decide to drink, always do so responsibly. Never drink and drive.

A Pictorial Description of Alcohol Consumption and Accumulation in the Stomach

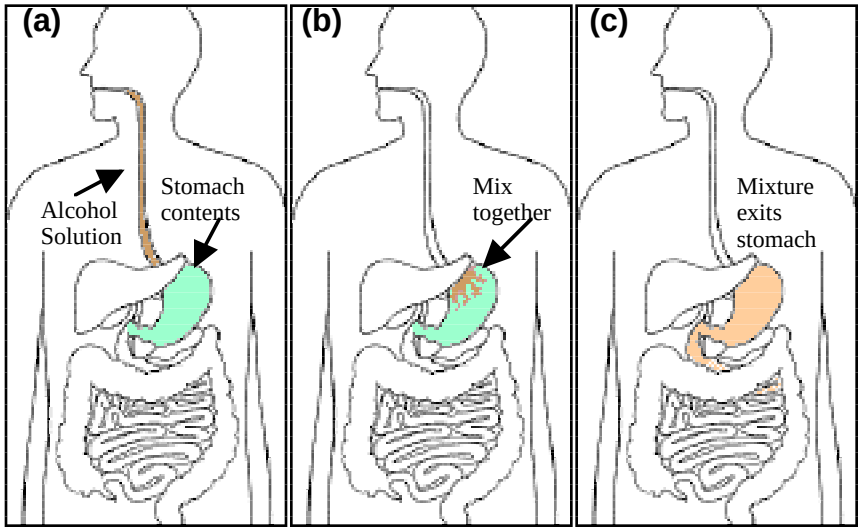

Figure 1: (a) The person drinks some alcohol solution (brown) which travels to the stomach. The current contents of the stomach are shown as a blue solution. (b) The alcohol solution enters the stomach and is mixed with the stomach contents. The resulting solution is shown in as a tan color. (c) Some of the resulting solution must leak out of the stomach (into the small intestine) to avoid bursting the stomach.

In this activity, you will be trying to use functions and derivatives to represent the accumulation of alcohol in a person's stomach. To do this realistically is very complicated and would involve (in addition to some complicated mathematics) expert knowledge of human anatomy, physiology and biochemistry. We are going to make a number of simplifying assumptions. Most of these assumptions are quite unrealistic, but they will make the math a *lot* easier.

The assumptions are:

- No water or alcohol are absorbed by the mouth, esophagus or stomach.
- The only way that water and alcohol can leave the stomach is through the small intestine (i.e. no vomiting or absorption across the stomach or gut walls).
- The only substances present in the stomach of the person are water and alcohol.
- Water and alcohol are not chemically changed during the entire process (in particular, there are no bodily enzymes such as *alcohol dehydrogenase* that break the alcohol down in the mouth or stomach).
- The percentage of the stomach contents that is alcohol determines a person's physiological response (i.e. their level of intoxication).

The process of imbibing alcohol is shown in Figure 1[1].

The Scenario that you will describe using Mathematics

Figure 2: (a) The person consuming the alcohol solution in the manner described by the scenarion. (b) A sample of the alcohol solution.

The scenario that you will try to describe mathematically is as follows:

- A person starts out with 4000 ml of water in their stomach[2].
- The person starts to drink (see Figure 2[3]) an alcohol solution. The solution consists of 43% alcohol and 57% water[4].
- The person drinks the alcohol solution at a rate of 240ml per hour[5].
- Because the stomach is fully expanded (to a volume of 4000 ml), the stomach contents will also leave the stomach (moving into the small intestine) at a rate of 240 ml per hour.

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[1] The outline of the human digestive system was obtained from: http://www.niddk.nih.gov/

[2] The volume of a human stomach is normally about 1000 ml (Source: J. Bevan. *Handbook of Anatomy and Physiology*. New York: Simon and Schuster, 1978.). However, the stomach is a very flexible organ and can expand to hold up to 4000 ml (Source: H. Curtis and N. S. Barnes. *Invitation to Biology. Fifth Edition*. New York: Worth, 1994.).

[3] Image source: http://www.postershop.com/

[4] Source: http://www.jackdanielsgoods.com/

[5] This is roughly equivalent to one shot glass every ten minutes.

Your Objective

Let T represent the amount of time (in hours) that the person has spent drinking the alcohol solution.

Let $A(T)$ represent the amount of alcohol (in ml) that are currently in the person's stomach.

Your objective is to write down an equation for $A'(T)$, the rate at which the amount of alcohol in the person's stomach changes. A potential starting point is given below.

rate of change of amount of alcohol in stomach	=	rate that alcohol enters stomach	–	rate that alcohol leaves stomach

When you have come up with this equation, you will use the equation to determine how quickly the symptoms of intoxication are likely to appear in the person shown in Figure 2.

- ***Use the space provided here to write down an equation for the rate of change of the amount of alcohol in the person's stomach.***

Analyzing the Predictions of the Model

In reality, the amount of alcohol that you have in your stomach does not determine your physiological response, it is the amount of alcohol that has entered your bloodstream that determines your response to alcohol.

The percentage of blood that is comprised of alcohol is called the Blood Alcohol Level (or BAL). Some typical effects of alcohol consumption are listed in Table 1[6] (below).

BAL (%)	Symptoms
0.01-0.05	Behavior normal in most subjects
0.03-0.12	Mild euphoria, sociability, talkativeness, decreased inhibitions, diminution of judgment and control. Loss of efficiency in performance tests.
0.09-0.25	Emotional instability. Impairment of perception and memory. Increased reaction time and decrease in sensory-motor coordination. Drowsiness.
0.18-0.30	Exaggerated emotional states. Decreased muscular coordination, staggering walk and slurred speech. Disorientation and mental confusion. Vomiting.
0.25-0.40.	Inertia and loss of motor functions. Marked decrease in response to stimuli. Inability to stand or walk. Vomiting and possible incontinence. Sleep or stupor.
0.35-0.50	Coma. Depressed reflexes. Subnormal body temperature. Impairment of circulation and respiration. Incontinence. Possible death.
0.45+	Death from respiratory arrest.

Table 1.

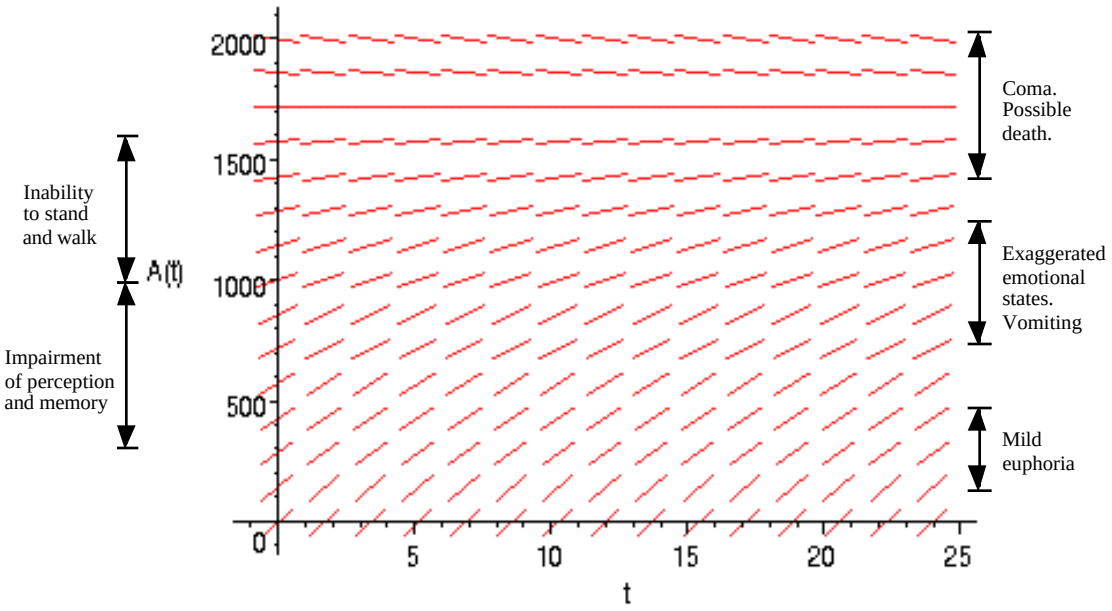
However, in our simplified model of alcohol consumption, we have assumed that the amount of alcohol in the stomach does determine your physiological response.

- ***The slope field shown on the next page corresponds to the differential equation:***

$$A'(T) = 103.2 - 0.06 \cdot A(T).$$

Use the slope field to determine when the person shown in Figure 2 will experience the physiological responses (symptoms) shown in Table 2 below.

[6] Source: K. M. Dubowski. "Stages of alcohol intoxication." Available on-line from: www.intox.com



After this many hours of drinking ...	The person shown in Figure 2 will exhibit these symptoms ...
	Mild euphoria, sociability, talkativeness, decreased inhibitions, diminution of judgment and control.
	Impairment of perception and memory. Increased reaction time and decrease in sensory-motor coordination. Drowsiness.
	Exaggerated emotional states. Staggering walk and slurred speech. Disorientation and mental confusion. Vomiting.
	Inability to stand or walk. Vomiting and possible incontinence. Sleep or stupor.
	Coma. Subnormal body temperature. Impairment of circulation and respiration.
	Death from respiratory arrest.

Table 2

A Precise Representation of Amount of Alcohol as a Function of Time

A slope field can give you estimated values for the solution of a differential equation, but it usually can't give you *exact* values for the solution of a differential equation. To do that, you would really need a formula for $A(T)$ that would allow you to work out the values of this function exactly.

By the end of Math Xb, you will be able to start with a differential equation and work out the formula of the function that is the solution of the differential equation. For now, however, all we expect of you is this: Given a differential equation and a formula for a function, you can decide whether the function really is a solution of the differential equation or not.

- ***The differential equation that describes the amount of alcohol in the person's stomach is:***

$$A'(T) = 103.2 - 0.06 \cdot A(T).$$

Both of the functions defined below have graphs with the same rough, overall shape predicted by the slope field for the equation $A'(T) = 103.2 - 0.06 \cdot A(T)$***. Decide which of these two functions actually represents a solution to the differential equation, by seeing which of the two functions "works" in the differential equation*** $A'(T) = 103.2 - 0.06 \cdot A(T)$**.**

Candidate 1: $A(T) = 860 \cdot \sqrt{T}$

Candidate 2: $A(T) = 1720 - 1720 \cdot e^{-0.06T}$