ICE - Integral Notation

Human beings are intensely visual creatures. For example, humans have even created visual representation systems for auditory phenomena such as music (see Figure 1[1]).

Figure 1: A sample of sheet music. (In this case, it is the beginning of the Canadian nation anthem.)

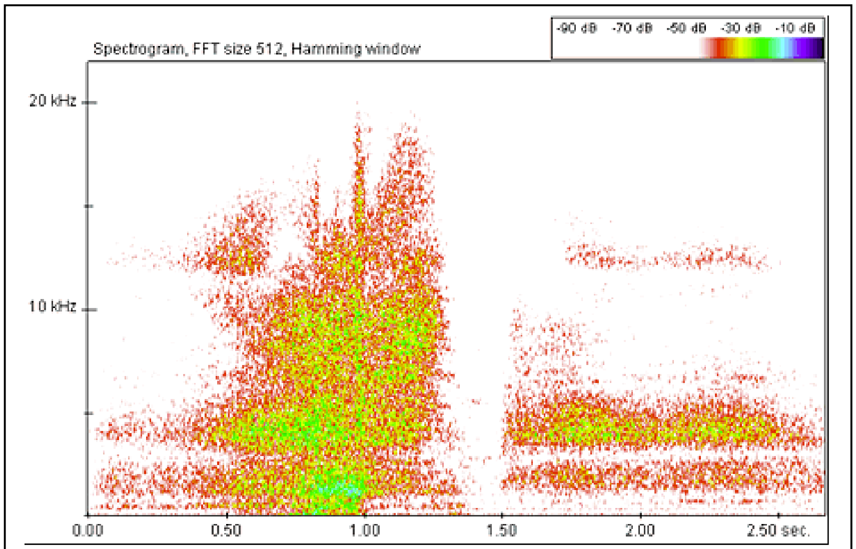


Figure 2: Spectrogram. A spectrogram is a graph showing the distinct frequencies that are present in a recorded sound at various times. In other words, a spectrogram is a graph showing frequency versus time. This particular spectrogram is called an “intensity spectrogram” as it has been shaded to reflect the intensity of each of the different frequencies of sound that are present.

The notation used for describing musical compositions is not the only form of visual representation that humans have created to describe sound. Figures 2[2] and 3[3] show two more complicated forms of representation – the spectrogram and power density spectrum, respectively.

To produce either a spectrogram or a power density spectrum, you begin with a noise. The noise is entered into a device called a “spectrum analyzer.” The spectrum analyzer can determine the different frequencies of sound that are present in the recorded noise, and the strength or intensity of each of the different sound frequencies that are present.

---

[1] Image source: http://www.pgh.gc.ca/ceremonial-symb/english/mus_e.html

[2] Image source: http://www.flmnh.ufl.edu/natsci/herpetology/brittoncrocs/croccomm.html

[3] Image source: http://www.flmnh.ufl.edu/natsci/herpetology/brittoncrocs/croccomm.html

The* spectrogram *is a plot showing the acoustic frequencies that are present at any particular time. The spectrogram is a plot of frequency (measured in hertz, Hz) versus time, which is measured in seconds in Figure 2.

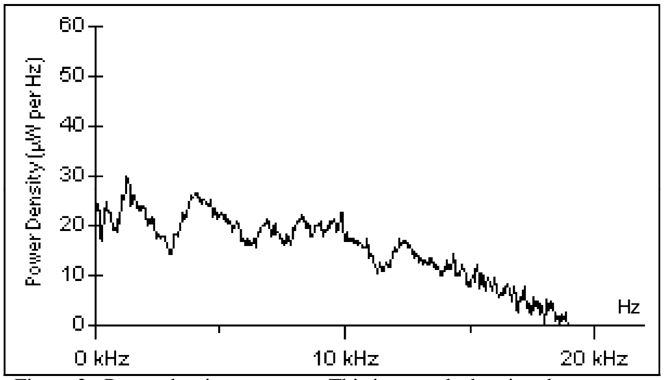


Figure 3: Power density spectrum. This is a graph showing the power density (in units of microwatts per hertz, μW per Hz) versus the distinct frequencies that are present in the recorded sound.

The* power density spectrum *is a plot of the intensity with which each distinct frequency is present in the sound. For example, a sound with a lot of bass will have a power density spectrum that is high for low frequencies. The power density spectrum of a sound is a graph of power density (measured in microwatts per hertz, μW/Hz) versus frequency (measured in hertz, Hz).

• ***What are the units of the area under a power density spectrum? What interpretation would you give to the value of the area under a power density spectrum?***

Figure 4: An adult alligator (*Alligator mississippiensis*) making a high intensity warning noise while displaying its impressive jaws and teeth.

The graphs shown in Figures 2 and 3 depict the high intensity warning noise of an alligator (Alligator mississippiensis – see Figure 4[4]). When an alligator makes this warning noise it is trying to communicate, "If you come any closer or do anything suspicious, I will bite you."

[4] Image source: http://www.tpwd.state.tx.us/expltx/urban/alligators.htm

In this ICE, you will use integral notation to describe and calculate some of the quantities that behavioral wildlife biologists routinely measure for a wide variety of animals including alligators. (Figure 3 is shown below for your convenience in answering the next few questions.)

Note that the small break in the spectrogram shown in Figure 2 (near the 1.5 second mark) was where the alligator stopped hissing for a moment and tried to bite the herpetologist who was goading the alligator into making the threat warning noise[5].

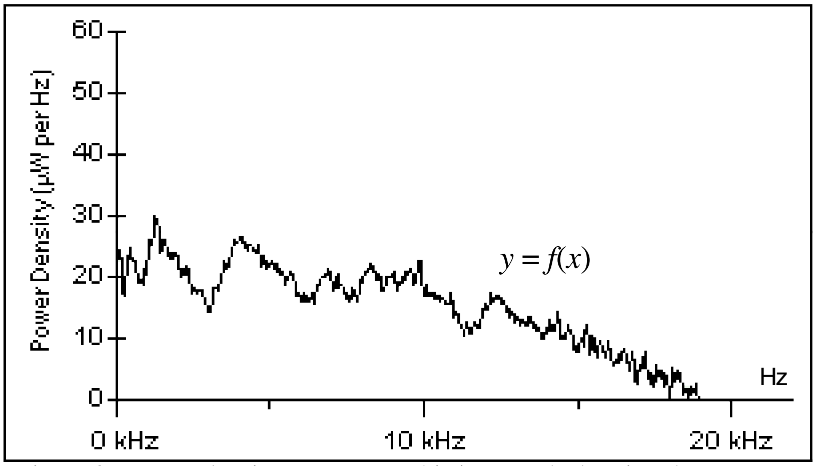


Figure 3: Power density spectrum. This is a graph showing the power density (in units of microwatts per hertz, μW per Hz) versus the distinct frequencies that are present in the recorded sound.

• ***How could you use integral notation to represent the total acoustic power of the alligator when it is making an intense-threat warning noise?***

• ***What kind of function would do a reasonable job of representing the trend shown in Figure 3? Some values of points on the graph of Figure 3 are given in Table 1 below. Enter these points into your calculator and find an equation for the function*** $f(x)$***.***

Frequency (hertz, Hz)	0	5	10	15	17.5
Power density (μW per Hz)	24	22	19	10	0

Table 1: Coordinates of some points from Figure 3.

[5] Fortunately, both herpetologist and alligator emerged from the data collection session unscathed.

• ***Using 200 rectangles, the equation the equation that you found for** $f(x)$ **and your graphing calculator, find the total acoustic power generated by the alligator when it makes the intense-threat warning noise. For comparative purposes, the acoustic powers[6] of some familiar sounds are given in Table 2 (below).***

Sound	Typical acoustic power (watts, W)
Normal speech	0.00004
Loud speech	0.0004
Loud music	0.004
Shouting	0.04
Your neighbor's stereo at 3 a.m.	0.4

Table 2: Acoustic power levels for common sound sources.

Figure 5: An alligator hatchling emerges from its leathery egg. Young alligators receive some of the best parental care of any reptile and are carefully guarded by their mothers. These young alligators normally eat insects, moving on to frogs and small fish as they grow.

Figure 5[7] shows a baby alligator hatching from an egg. Despite their fearsome appearance, adult female alligators provide a very high level of care for their young and respond rapidly to the distinctive "distress calls" that baby alligators make.

A baby alligator's distress call usually consists of a string of short "yelps." The power density spectra for two consecutive "yelps" are shown in Figures 6 and 7[8].

---

[6] Source: http://ww.jwdavis.com/tech/notes/TECH%20NOTE-acoustic.pdf

[7] Image source: http://www.tpwd.state.tx.us/expltx/urban/alligators.htm

[8] Image source: http://www.flmnh.ufl.edu/natsci/herpetology/brittoncrocs/croccomm.html

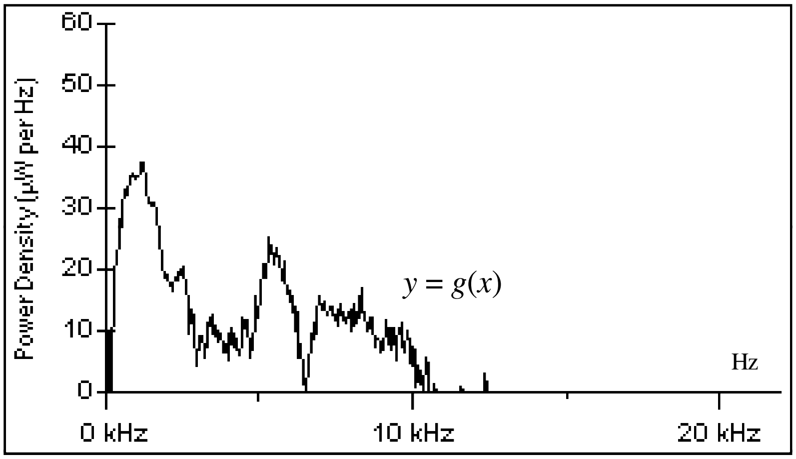


Figure 6: Power density spectrum for the first yelp in a hatchling alligator's distress call.

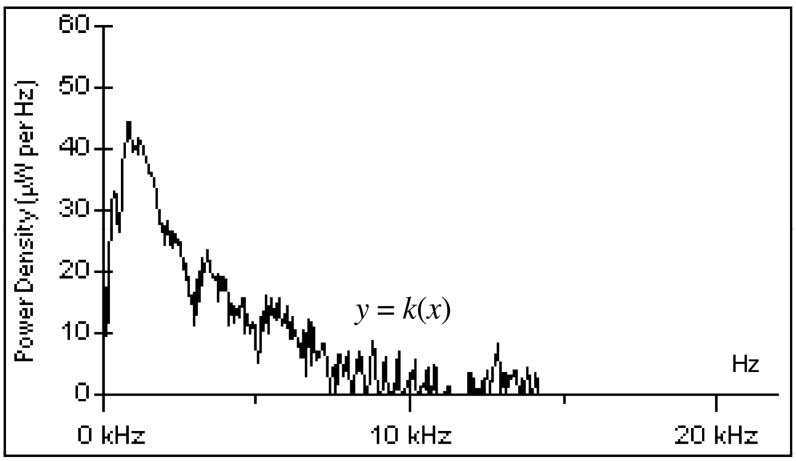
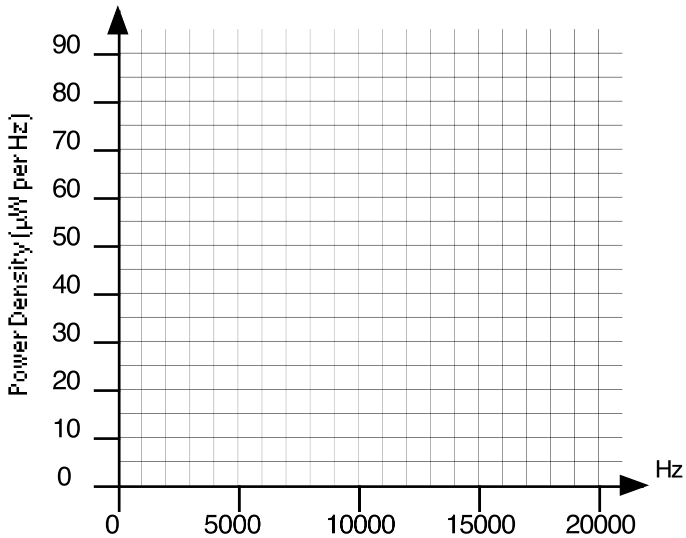


Figure 7: Power density spectrum for the second yelp in a hatchling alligator's distress call.

- ***Use integral notation to express the acoustic power produced by the hatchling alligator during each of the two "yelps" shown in Figures 6 and 7.***

- ***Use the axes provided (together with the graphs shown in Figures 6 and 7) to sketch the total power density spectrum of both yelps combined. What is the equation (expressed in terms of $g(x)$ and $k(x)$) for the total power density spectrum?***



• ***How could you express the total acoustical power of both yelps using integral notation?***

Figure 8: A group of adult alligators bask in the shallow water at the edge of a lake.

Figure 8[9] shows a group of alligators basking on the side of a lake. Figure 9[10] shows the power density spectrum of the low-intensity warning noise of an adult alligator. (This is the noise that an alligator might make if it were trying to communicate, "Don't come any closer.")

[9] Image source: http://www.sufari.net/~pricetag/David/trip.html

[10] Image source: http://www.flmnh.ufl.edu/natsci/herpetology/brittoncrocs/croccomm.html

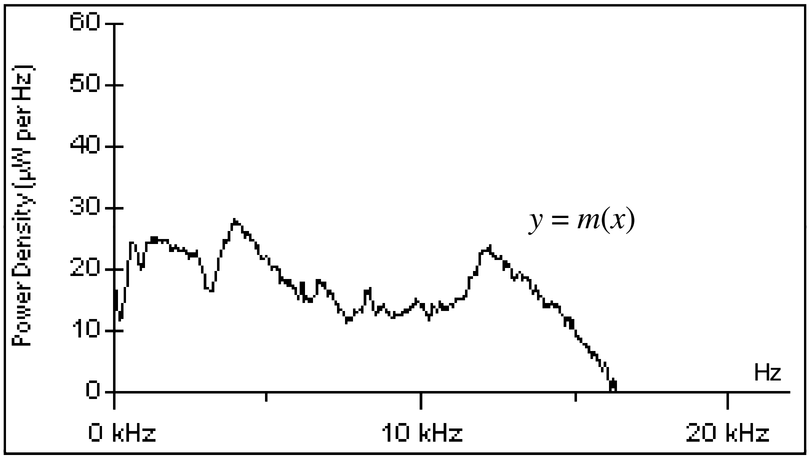


Figure 9: Power density spectrum for the low intensity warning noise of a single adult alligator.

• ***Describe a transformation (horizontal stretch, vertical shift, etc.) of the graph of $y = m(x)$ that would transform the given power density spectrum into the power density spectrum of the entire group of alligators.***

• ***How could you express the total acoustic power of the entire group of alligators shown in Figure 8 if they all made low-intensity warning noises in unison? (As they might if approached by a person in plain sight.)***