ICE - Functions Defined by Integrals

The fastest man in the world is currently Maurice "The Kansas Cannonball" Green (see Figure 1[1]). In 1999, Mr. Green broke the world record in the men's 100m, with a time of 9.79s. Figure 2 shows the velocity-time graph for a race that Mr. Green won at the 1997 World Championships in Athens, Greece.

Figure 1: Maurice "The Kansas Cannonball" Green.

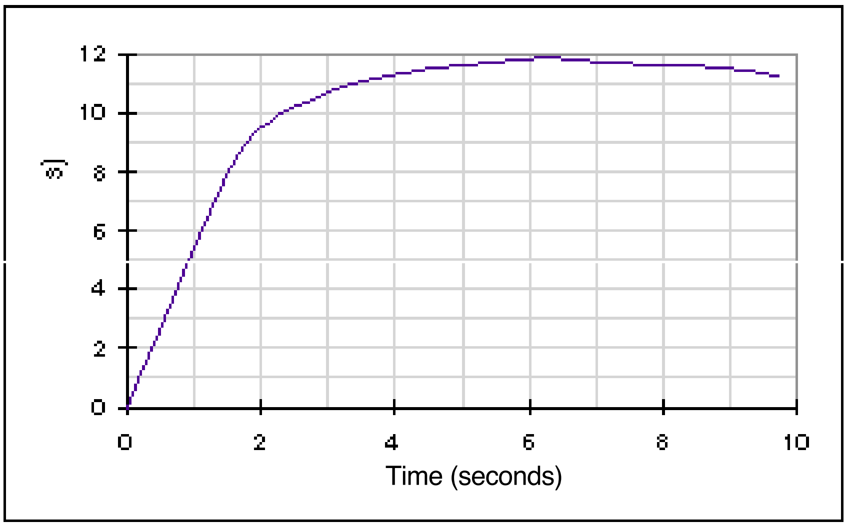


Figure 2: Velocity-time graph for Maurice "The Kansas Cannonball" Green during the 1997 World Track and Field Championships.

- ***What are the units of the area under the graph in Figure 2? What interpretation does this suggest for the area under the graph in Figure 2?***

- ***Approximate the total area under the graph in Figure 2.***

- ***What athletic event is probably represented by Figure 2?***

[1] Image Source: http://www.pulsecheck.com/

- ***Can you be absolutely sure of your previous answer? If not, what additional information would you need in order to be sure of your previous answer?***

The graph shown in Figure 2 is quite accurately represented by the cubic function:

$$v(x) = 0.046 \cdot x^3 - 0.925 \cdot x^2 + 5.811 \cdot x + 0.369.$$

- ***Find an equation for the antiderivative of $v(x)$. What interpretation can you give to the constant "$+C$" in your antiderivative formula?***

- ***Use your antiderivative formula to find the numerical values indicated below.***

a	b	$\int_a^b v(x) \cdot dx$	*Interpretation*
1	5		
2	8		
0	9.73		

- ***Even though you don't know the value of the constant "$+C$" in your antiderivative formula, you should have been able to find precise numerical values for symbolic expressions like:***

$$\int_1^5 v(x) \cdot dx.$$

Why is the numerical value of the integration constant "$+C$" not relevant when you are finding the numerical value of a definite integral?

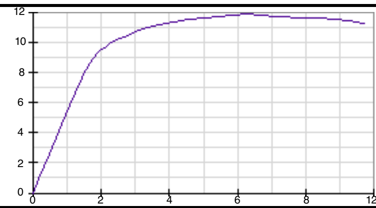
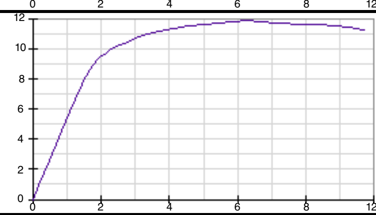
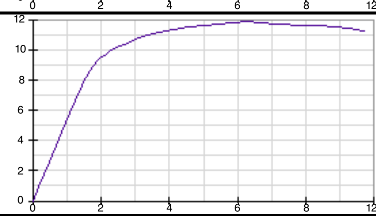
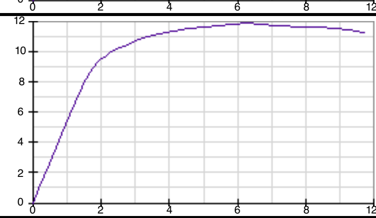
The collection of symbols:

$$s(T) = \int_1^T v(x) \cdot dx$$

defines a new function, $s(T)$, as an integral of the velocity function $v(x)$. Note that the independent variable of the function $s(T)$ is T whereas the independent variable in the function $v(x)$ is different – the independent variable in the function $v(x)$ is x.

- ***Based on the calculations that you have already done, how would you interpret the outputs generated by the new function $s(T)$?***

- ***Use the definition of the new function, $s(T)$, to complete the entries in the table given below.***

Quantity	Verbal description	Graphical description	Numerical value
$s(5)$			
$s(9.7)$			
$s(1)$			
$s(0)$			

- ***What is the derivative of the function $s(T)$? Try to answer this question in two ways. First, explain in a sentence or two what the derivative of the function $s(T)$ represents in terms of position, velocity and Maurice "The Kansas Cannonball" Green. Second, try to find a formula for $s'(T)$.***