ICE - Geometric Series

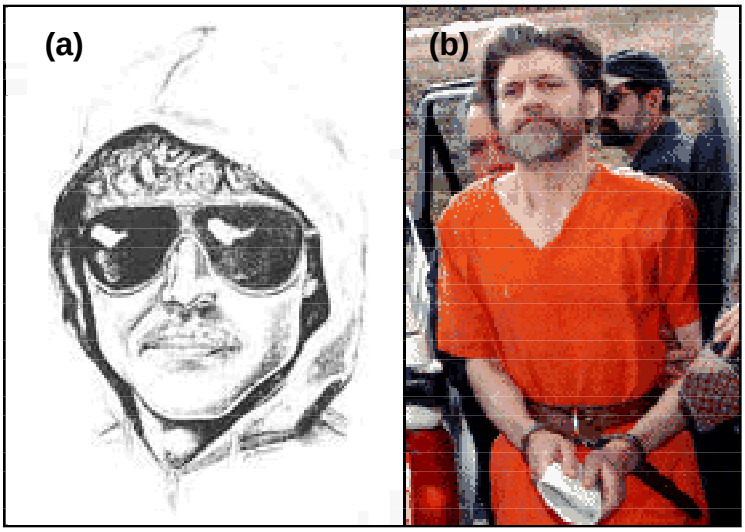

Figure 1(a,b): Artist's Conception and actual appearance of Theodore John Kaczynski alias "The Unabomber." Kaczynski is a graduate of both Harvard University and The University of Michigan, and was Assistant Professor of Mathematics at The University of California, Berkeley from 1968-1971.

Occasionally, people decide that they are dissatisfied with life in high-stress, high-technology urban environments and long for a simpler and more peaceful existence in a pleasant natural setting (see Figure 1[1]).

Property costs in rural parts of the United States are often significantly lower than urban areas. For example, a property recently advertised near Houlton, ME (see Figure 2[2]) offered 56 acres of former farm land for $28,700.

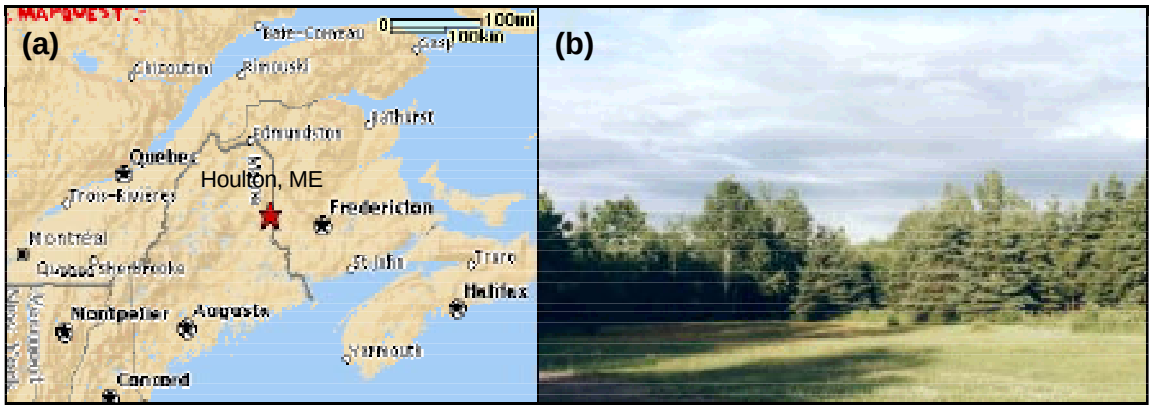

Figure 2: (a) Location of Houlton, ME, near the Canadian border. (b) A section of the 56 acre property offered for sale.

[1] Image sources: http://www.crimelibrary.com and http://www.pub.umich.edu
[2] Image sources: http://www.mapquest.com and http://meadowsandmountains.com/farm.htm

Figure 3: Artist's rendition of the "Hampshire" log cabin manufactured by Granite State Log Homes, Inc.

Rural properties are usually undeveloped, however, and do not often feature a structure that is suitable for year-round human habitation. Figure 3[3] shows one option for housing that can be reasonably easily assembled in a remote area - the log cabin. The particular cabin pictured in Figure 3 is manufactured by the Massachusetts based company "Granite State Log Homes." The basic kit[4] retails for $53, 173.

Estimates[5] of all of the costs associated with constructing a log cabin on a rural property (like the one shown in Figure 2) are shown in Figure 4 (below).

Maine Fantasy

	A	B	C
1			
2	Property	28700	
3	Cabin - kit	53173	
4	Transportation	9800	
5	Labor	22000	
6	Misc. costs	7505	
7	Local taxes	4722	
8			
9			
10			

Sheet1 Sheet2

Figure 4: Costs associated with locating a "Hampshire" log cabin on a 56 acre property in rural Maine.

- ***A disgruntled math professor can afford to put down $28,000 towards the purchase of his fantasy house (a log cabin in Maine). About how much would this disgruntled math professor need to borrow in order to make his fantasy come true?***

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[3] Image source: http://www.loghomes.com/GraniteStateLogHomes/yeararound.html

[4] Source: http://www.loghomes.com/GraniteStateLogHomes/yeararound.html The model pictured in Figure 3 is the "Hampshire."

[5] Source: http://www.loghomes.com/

Table 1[6] gives the current[7] mortgage rates for the Massachusetts-based lending company "Ocean West Funding."

Program Name	Annual Interest Rate (%)	Maximum Loan	Term
30 Year Fixed	6.50	$300,000	30 years
15 Year Fixed	6.00	$300,000	15 years
30 Year Jumbo	7.00	$1,000,000	30 years
No Income Verification	8.25	$1,000,000	Variable

Table 1: Current Mortgage Rates for Ocean West Funding.

- ***Of the programs that Ocean West offer, which make the most sense for the math professor to consider as mortgage providers?***

Choose one of the programs to investigate further. Your main job for the rest of this ICE is to determine how large the math professor's monthly mortgage payment will be.

- ***Let the symbol M represent the professor's monthly mortgage payment in dollars. Use Table 2 (below) to work out how much the professor still owes the lender as time goes by. As you do this, try to find patterns in the terms that appear as you fill in the table.***

Time	***Amount math professor still owes ($)***
Loan approved	
1 month after loan approved	
2 months after loan approved	
3 months after loan approved	
4 months after loan approved	

Table 2: Use this table to work out how much the math professor will owe as time goes by.

[6] Source: http://www.hsn.com/lshow/ocean_west.html

[7] As of 2/13/2002. Rates are subject to change.

- ***Depending on which lending program you have chosen to study, the mortgage will be completely paid off in either 15 or 30 years. By following the patterns established in Table 2, write down an expression that represents the amount that the professor will owe after either 15 or 30 years.***

- ***Use the expression that you obtained above to calculate*** M ***the professor's monthly mortgage payment. One piece of information that might be useful to you when you carry out this calculation is the summation formula for a geometric series:***

$$a + a \cdot r + a \cdot r^2 + \ldots + a \cdot r^n = \frac{a \cdot \left(1 - r^{n+1}\right)}{1 - r}.$$