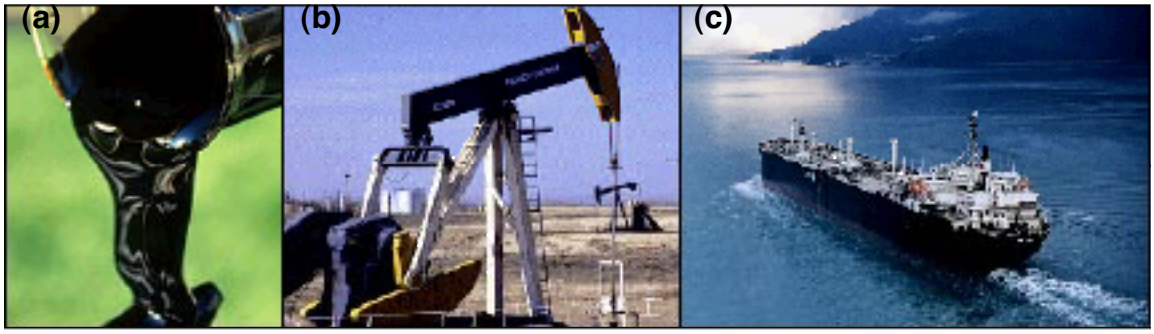
ICE - Calculating Antiderivatives

Petroleum is an integral part of the US economy and lifestyle. Figure 1[1] illustrates the process by which crude oil is extracted from the earth, transported around the globe and eventually refined into consumer and industrial products.



Figures 1(a, b, c): (a) Crude oil is a thick, black substance that consists of a mixture of important hydrocarbons and organic impurities. (b) Crude oil is pumped from a well in an established oil field. (c) The crude oil is then transported around the globe in large tanker ships.

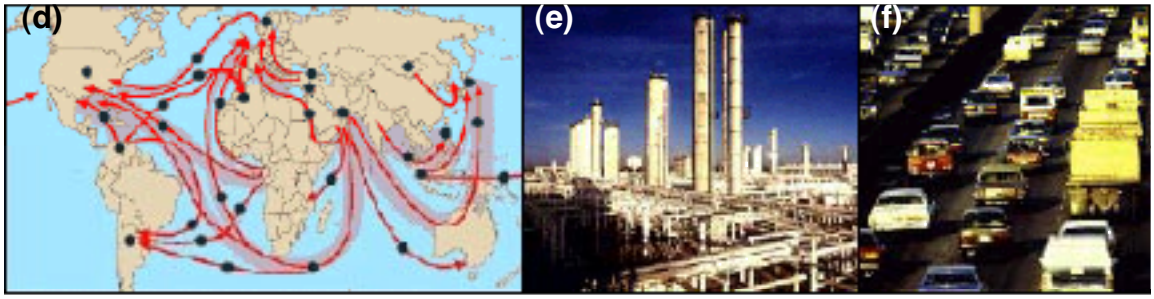


Figure 1(d, e, f): (d) From oil fields located in only a few parts of the world, crude oil is exported over the entire globe. (e) When the crude oil has been delivered to its destination it is refined to extract the commerically valuable compounds. This image shows a major oil refinery located in New Jersey. (f) Finally, the refined petroleum products are used by industry and private citizens.

In this ICE you will use your abilities to fit functions to data and your knowledge of calculus to find an equation for the amount of petroleum remaining (as a function of time) in the whole world. You will then use this function to estimate when the known world reserves of petroleum will be depleted.

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[1] Image sources: http://www.syncrude.com/
http://www.howstuffworks.com/
http://maguireenergy.cox.smu..edu/
http://www.petrostrategies.org/
http://www.magnapatch.com/

• ***Table 1[2] (below) shows the instantaneous rate of the world petroleum reserves over the last few years. The units of the instantaneous rate are billions of barrels of oil per year. Enter the data into your calculator and find the equation of the linear function that best fits the data. Record your equation below.***

Year	Instantaneous Rate of Change of World Petroleum Reserves (Billions of barrels per year)
1998	−27.013
1999	−27.554
2000	−28.105

Table 1.

• ***Suppose that*** T ***represents the number of years since 2000 and the function*** $P(T)$ ***represents the world petroleum reserves (in units of billions of barrels of petroleum). Re-write the equation that you found with your calculator so that you can use it to complete the equation given below.***

$$P'(T) =$$

• ***Based on the equation that you have just written down for the derivative*** $P'(T)$, ***what would the equation for the function*** $P(T)$ ***have to look like? Write your equation for the function*** $P(T)$ ***below.***

NOTE: **1.** Remember that the derivative of a constant is equal to zero.

2. It is fine if your equation for $P(T)$ contains an unspecified constant.

[2] Source: U.S. Department of Energy, Energy Information Administration. "International Energy Outlook." 1999.

• ***According to petroleum giant BP-Amoco[3], the total known world petroleum reserves at the beginning of the year 2000 was 1033 billion barrels of energy. Use this information to calculate the value for the unknown constant in your equation for*** $P(T)$. ***When you have done this, write your equation for*** $P(T)$ ***here.***

• ***According to the equation that you have found for*** $P(T)$, ***when will the world's known reserves of petroleum be depleted? Show the details of your calculations and write your answer below.***

• ***A second estimate[4] of the world's petroleum reserves at the beginning of the year 2000 is 1950 billion barrels. If you use this larger estimate, when will the petroleum be depleted?***

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[3] Source: BP-Amoco. "Statistical Review of World Energy." 2000.

[4] This is also from the BP-Amoco study. This estimate is based on calculations that include oil reserves that might exist but have yet to be discovered. As you might imagine, this figure is somewhat controversial.

One criticism that could be made of the calculations that you have just performed is that you used a linear equation to create the equation for $P'(T)$. ***If you look very hard at the data, it is possible to see a hint of concavity in the data, suggesting that some kind of exponential function might do a better job of representing the trend in the data. If you fit an exponential function to the data then the equation[5] for*** $P(T)$ ***becomes:***

$$P(T) = 2452.26 - 1419.26 \cdot (1.02)^T.$$

- ***According to this new equation for*** $P(T)$, ***when will the world's known reserves of petroleum be depleted? Show the details of your calculations and write your answer below.***

NOTE: The US government views access the petroleum as a vital national security issue (see the Appendix hereafter for a discussion). Given this, the calculations that you have just performed strongly suggest that in our lifetimes there will be major global conflict over natural resources such as petroleum.

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[5] This equation uses the less controversial figure of 1033 billion barrels of petroleum for the world petroleum reserves at the beginning of the year 2000.

Appendix: US Foreign Policy, National Security and Access to Petroleum

Figure 2: The former Soviet republic of Kazakhstan.

On September 5, 1997, approximately 500 American soldiers parachuted into a desert region near the Tien Shan mountains in southern Kazakhstan[6] (see Figures 2[7] and 3[8]). This force was commanded by veteran Marine Corps General John Sheehan (see Figure 4[9]).

The mission of this heavily armed force was to link up with friendly forces from the former Soviet republics of Kazakhstan, Kyrgystan and Uzbekistan and repel renegade forces from the region.

Figure 3: U.S. soldiers of the 82nd Airborne division parachute from a C-141 Starlifter aircraft into Kazakhston on September 5, 1997.

Figure 4: U.S. Marine Corps general and commander of U.S. forces during CENTRAZBAT 97 John C. Sheehan.

Fortunately, this operation was only an exercise - part of Operation CENTRAZBAT '97. However, this was arguably one of the most unusual exercises that the US military has ever engaged in. The exercise began with the longest airborne operation in the history of warfare. Soldiers of the 82nd Airborne division flew from Fort Bragg, North Carolina, to southern Kazakhstan (a trip of approximately 7700 miles). The exercise was also the first deployment of US ground troops in a former Soviet republic, and the first example of direct US military cooperation with a country bordering the Caspian Sea (see Figure 2).

The extraordinary nature of this undertaking prompts the question: Why was the US military and government suddenly so interested in cooperating with a former enemy nation that has little in common (culturally or economically) with the United States?

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[6] Source: R. J. Smith. "U.S. leads peacekeeping drill in Kazakhstan." *Washington Post*, September 15 1997.

[7] Image source: http://www.khazaria.com

[8] Image source: http://www.usd.edu/~thompson/

[9] Image source: http://www.usd.edu/~thompson/

At the press conference announcing the successful completion of Operation CENTRAZBAT '97, Assistant Secretary of Defense Catherine Kelleher and other Pentagon officials maintained that their only objective was to demonstrate the support of the United States for the former Soviet republics and to encourage their continuing stability. Ms. Kelleher noted, "What we need here are independent, sovereign states that are able to defend themselves."[10]

Given this position from the government, it is difficult to understand why Operation CENTRAZBAT '97 was conducted in the way that it was. If the sole reason for providing military cooperation is to enhance the capability of the former Soviet republics to defend *themselves*, then it is difficult to explain why US ground troops were performing maneuvers to *augment* the military capability of the republics. Similarly, the US military has shown little interest in performing similar cooperative exercises with other former Soviet republics.

Many observers[11] believe that the interest shown by the US government in the states bordering the Caspian sea has more to do with geological surveys indicating the presence of enormous reserves of oil and natural gas in the region[12]. Reports from the US Department of Energy[13] suggest that the nations bordering the Caspian sea (Azerbaijan, Kazakhstan, Turkmenistan and Uzbekistan - see Figure 2) currently have approximately 20% of the known world reserves of oil and perhaps as much as 12% of the currently known world reserves of natural gas. The only region on earth with a greater abundance of petroleum is the Persian Gulf, which has about 63% of the known world reserves of petroleum[14].

Figure 5: Former Secretary of State James Baker.

Access to energy reserves in the form of petroleum has a major priority of the US government for many decades. James Baker (see Figure 5[15]) was Secretary of State during the presidency of George H. W. Bush. In a recent interview with the television program "Frontline," Mr. Baker stated:

> "...I worked for four administrations under three presidents. And in every one of those, our policy was that we would go to war to protect the energy reserves in the Persian Gulf. That is a major and very significant national security interest that we have." - former Secretary of State, James Baker.

Figure 6 shows a time-line of how US policy regarding access to petroleum reserves (especially the petroleum reserves of the Persian Gulf) has evolved over the last 70 years[16].

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[10] Source: R.J. Smith. "U.S. leads peacekeeping drill in Kazakhstan." *Washington Post*, September 15 1997.

[11] Source: M.T. Klare. "Resource Wars. The New Landscape of Global Conflict." New York: Metropolitan Books, 2001.

[12] You may have inadvertently learned about the existence of these reserves if you saw the latest James Bond film, "The World is Not Enough." The character played by Sophie Marceau was the heiress to a fortune based on Caspian sea oil.

[13] Source: U.S. Department of Energy, Energy Information Administration. "Caspian Sea Region." June 2000.

[14] Source: BP-Amoco. "Statistical Review of World Energy." 1999.

[15] Image source: http://www.cnn.com The quote is from a transcript located at: http://www.pbs.org/

[16] Source: M.T. Klare. "Resource Wars. The New Landscape of Global Conflict." New York: Metropolitan Books, 2001.

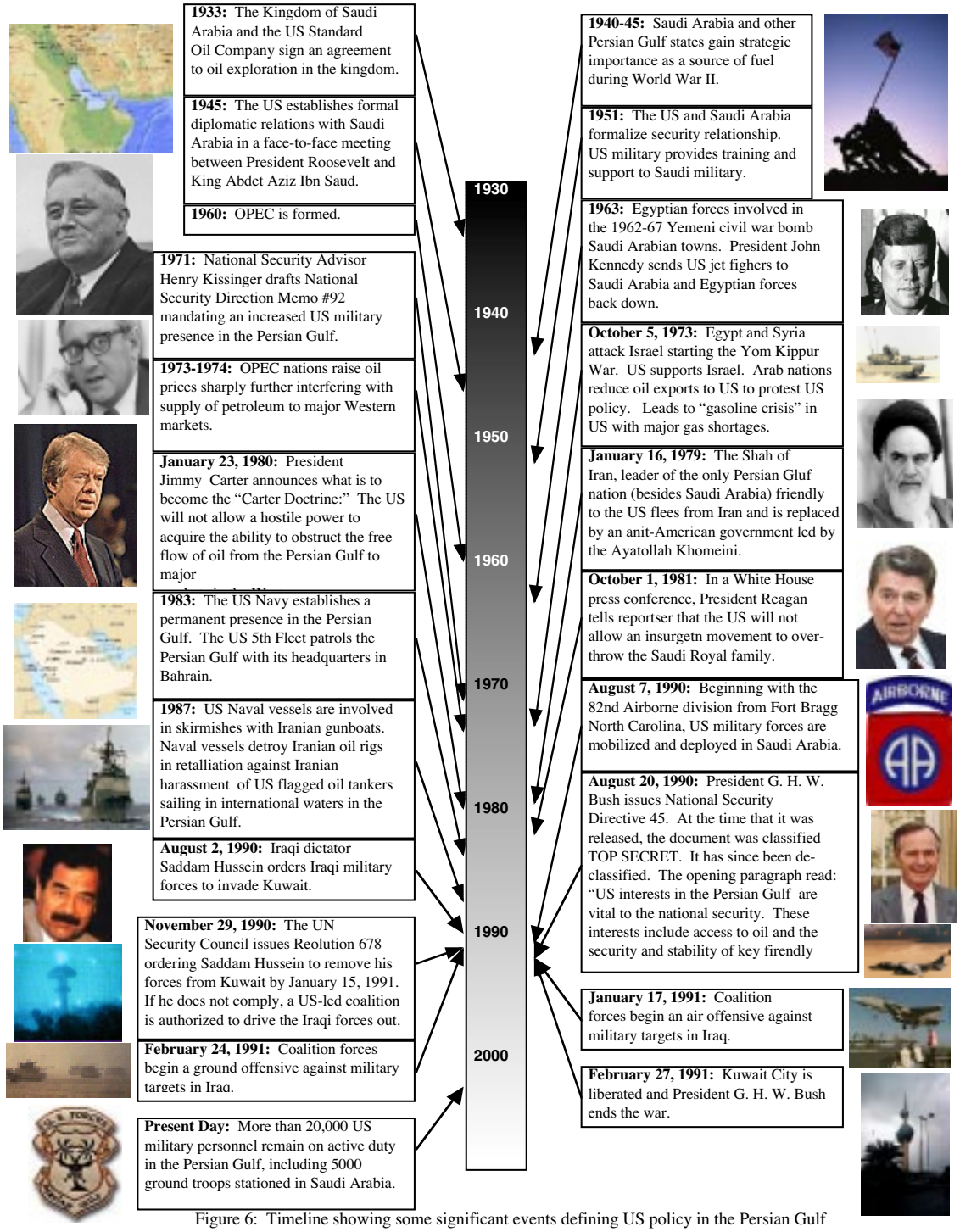

Figure 6: Timeline showing some significant events defining US policy in the Persian Gulf

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Image sources: www.mideastnews.com, iwo-jima-memorial.visit-washington-dc.com, FDR Library, JFK Library, US Air Force, Jimmy Carter Library, www.salon.com, Ronald Reagan Library, www.foxnews.com, www.ahram.org.eg, US Navy, US Army, www.eskimo.com, www.greatdreams.com, CNN, United Nations, George H. Bush Library.