



ICE - Area Under a Derivative Graph



Figure 1: Layered with lettuce, onion, tomato and smothered with premium smoked Atlantic salmon this bagel is a sandwich-lover's delight.

On an average day approximately 24,000 people die from hunger worldwide¹. In the time that it took for you to read that sentence two or three people died as a result of hunger-related medical, physical or nutritional problems². Globally, approximately 841 million people suffer from malnutrition³. Although sobering, these figures are a huge improvement over the situation in the 1970's when approximately one third of the world's population suffered from malnutrition and hunger. Experts⁴ are quick to point out that this is not so much a problem of the amount of food produced in the world, but how that food is distributed. The Food and Agriculture Organization of the United Nations⁵ has estimated that the currently world agricultural production could satisfactorily feed a population equal to about 115% of the current world population.

An important fact to bear in mind when considering problems of global hunger and malnutrition is that around the world, agricultural production is profit-driven. That is, agricultural producers tend to favor crops and animal products that will yield a large profit. As such, a great deal of research (but not all⁶) is directed towards highly profitable agricultural products.

¹ Source: World Health Organization. <http://www.who.int/>

² Only a small percentage of hunger-related deaths are due to actual starvation. The vast majority of deaths are due to deficiencies of fundamental nutritional requirements (such as vitamins and minerals) and the accompanying diseases². Although it is tempting to think of these problems as confined to the developing world, some experts² estimate that as many as 30 million Americans (between 10% and 13%) of the population suffer from malnutrition.

³ Source: Food and Agriculture Organization of the United Nations. <http://www.fao.org/>

⁴ Source: P. Ulvin. (1994) "The State of World Hunger." Providence, RI: World Hunger Program, Brown University.

⁵ Source: Food and Agriculture Organization of the United Nations. (1993) "The State of Food and Agriculture." Rome, Italy: United Nations.

⁶ An important example of agricultural research that was not directed only towards increasing the profitability of agricultural production is the work of Nobel Laureat Norman Borlaug. Dr. Borlaug was awarded the Nobel Peace Prize in 1970 for his work on maize and wheat crop improvement. Dr. Borlaug's work ultimately lead to the so-called "Green Revolution."

An example of the influence of profit-directed research efforts is marine aquaculture. “Farming the ocean’s bounty” was seen at one time as a highly viable solution for the world’s hunger and malnutrition problems. However, two areas of aquaculture that have received a good deal of research interest are the farming of (luxurious and highly profitable) shrimp and salmon (see Figure 1⁷).

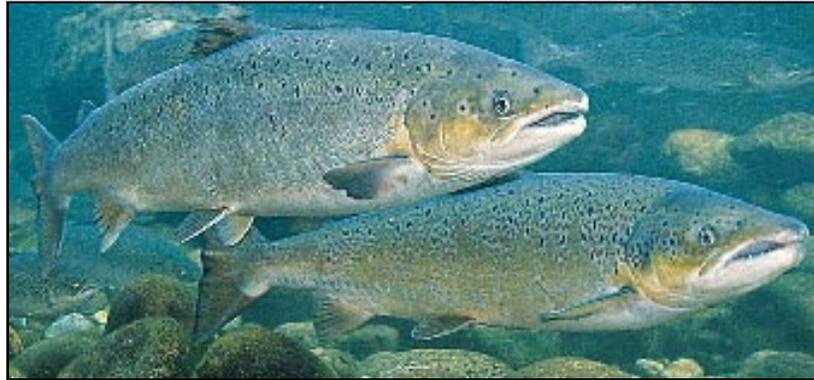


Figure 2: Adult Atlantic salmon (*Salmo salar*) in their natural environment.

One area of research that has been carried out with the highly sought-after (and increasingly depleted⁸) Atlantic salmon (*Salmo salar*, see Figure 2⁹) is the genetic modification (or genetic engineering) of fast-growing strains of salmon¹⁰. Genetic modification is a technique in which part of the genetic code of an existing creature is replaced with unrelated genetic material (see Figure 3¹¹).

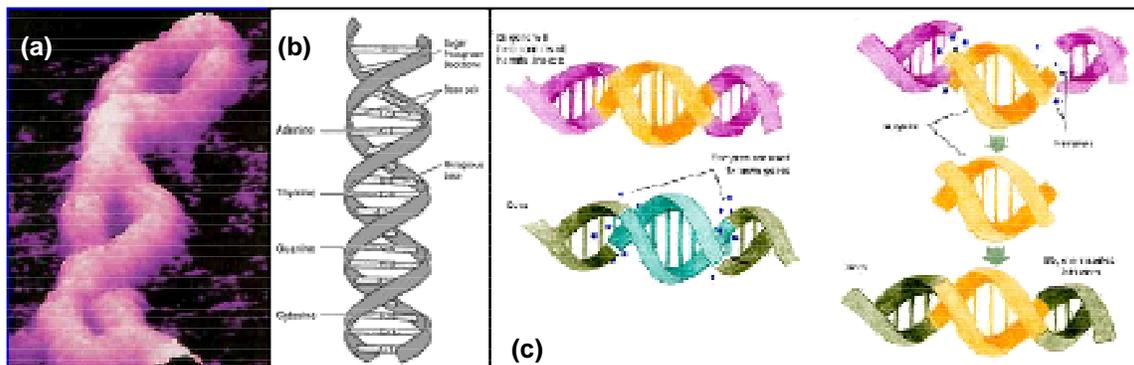


Figure 3: (a) At extreme resolution, DNA (deoxyribose nucleic acid) forms a molecule with the rough appearance of long strings of genetic material. (b) The standard model for the DNA molecule. DNA is thought to consist of a pair a long strong composed mainly of phosphorous and sugars. These long strands are joined (like a twisted ladder) by “rungs” that consist of the compounds adenine, guanine, cytosine and thymine. (c) Schematic showing the process of producing recombinant DNA. DNA is “cut” out of a donor and inserted into an existing DNA sequence.

⁷ Image source: <http://atlanta.creativeloading.com/>

⁸ Source: Maine Department of Conservation. <http://www.state.me.us/doc/dochome.html>

⁹ Image source: <http://www.geocities.com/Yosemite/Falls/6185/salmon2.htm>

¹⁰ Source: Aqua Bounty Farms, Inc. Nova Scotia, Canada.

¹¹ Image sources: (a) <http://www.time.com/time/daily/specials/genetics.htm>

(b) http://whyfiles.org/126dna_forensic/3.html (c) <http://www.ces.ncsu.edu/>

In the late 1990's researchers at the Canadian company Aqua Bounty, Inc. successfully inserted some of the genetic material from the Arctic Charr (*Salvelinus Alpinus*) into the DNA of Atlantic salmon. Atlantic salmon naturally inhabit the cold waters of the North Atlantic. However, they are not specifically adapted for life in cold waters. As a result, Atlantic salmon normally only grow during the summer months (normally June, July and August) when water temperatures are higher¹². As a species specifically adapted for life in frigid Arctic waters, the Arctic charr has a kind of natural "anti-freeze" (glycoproteins) that allows the fish to continue to grow even when the temperature of its habitat is very low.

The Canadian researchers were interested in identifying the charr genes that controlled the production and secretion of the "antifreeze" glycoproteins, and inserting these genes into the commercially valuable Atlantic salmon. In theory, this would produce a salmon that could continue to grow throughout the years, instead of just during the summer months.

The results were dramatic and are shown in Figures 4 and 5¹³ below. The



Figure 4: Juvenile Atlantic salmon (*Salmo salar*) in a laboratory aquarium. The small striped fish are natural salmon with no artificial genetic modifications. The large silver fish in the middle is a genetically modified salmon. The natural and genetically modified salmon pictures are *exactly the same* age and yet the genetically modified salmon is considerably larger than the unmodified fish.

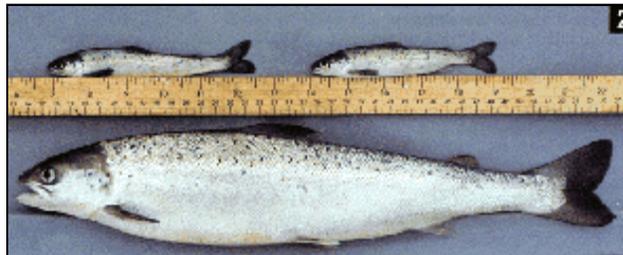


Figure 5: Farmed Atlantic salmon (*Salmo salar*) at two years of age (sub-adults). The salmon above the ruler are natural, unmodified salmon. The fish beneath the ruler is a genetically modified salmon. All of the fish pictured are exactly the same age.

genetically modified salmon grew at a much faster rate than the unmodified salmon. In this ICE you will use graphs of salmon growth rate to study just how much of a difference the genetic modifications made.

Tables 1 and 2¹⁴ give the growth rates for normal and genetically modified salmon during their first 18 months of life outside the salmon hatchery.

¹² This is phenomenon of summer growth does not appear to be linked to other conditions such as availability of food. Many of the adult salmon's natural habitats are no more productive (in terms of food items suitable for an adult salmon) in the summer.

¹³ Image source: <http://www.aquabounty.com/>

¹⁴ Source: C. Clarke. (1998) "Seawater growth of Atlantic salmon, *Salmo salar*." *Aquaculture Update*, **81**: 1-2. and J.A. Hutchings and M.E.B. Jones. (1998) "Life history variation and growth rate thresholds for maturity in Atlantic salmon, *Salmo salar*." *Canadian Journal of Fisheries and Aquatic Sciences*, **55**(1): 22-47.

(The life cycle of a salmon is shown in Figure 6¹⁵ below. The salmon are normally transferred from a hatchery to large tanks or ocean enclosures (see Figure 7¹⁶) when they are smolts.)

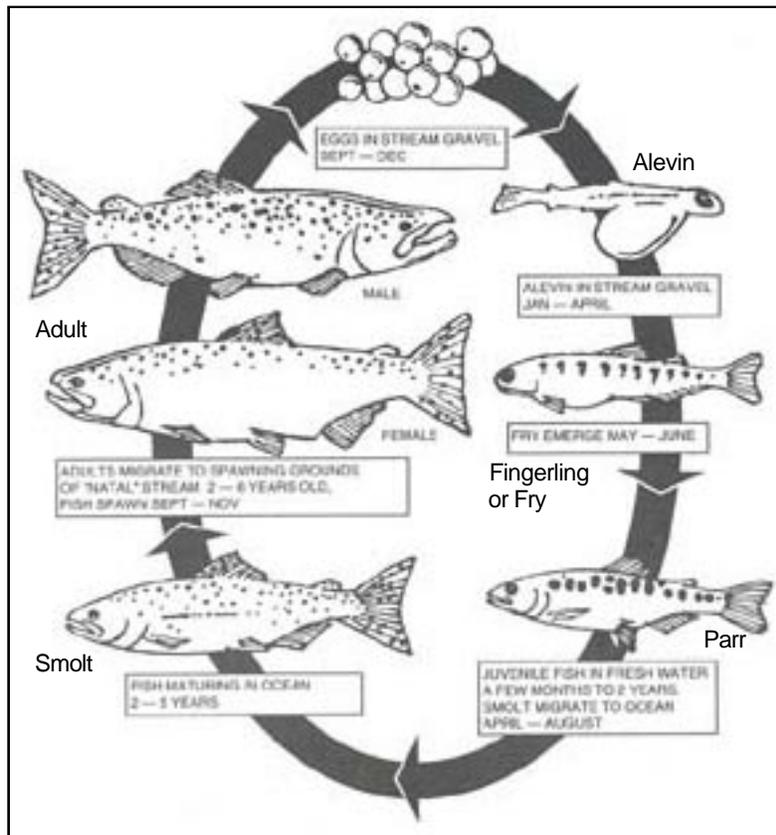


Figure 6: Life cycle of the Atlantic salmon (*Salmo salar*)

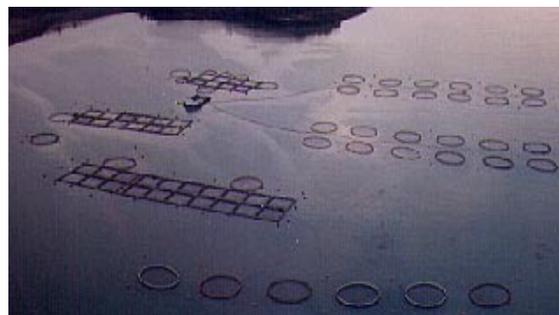


Figure 7: Salmon grow-out pens at Majestic Salmon Farms, ME. This facility does not raise genetically modified salmon.

¹⁵ Image source: http://www.jsd.k12.ak.us/ab/jones_cl/1cycle.htm

¹⁶ Image source: <http://www.majesticsalmon.com/>

Time since release (months)	0	3	6	9	12	15	18
Growth Rate (g/month)	6.061	8.289	11.336	15.504	21.203	29.998	39.659

Table 1: Growth Rate for Normal Atlantic Salmon, *Salmo salar*.

Time since release (months)	0	3	6	9	12	15	18
Growth Rate (g/month)	24.243	33.156	45.345	62.015	84.814	115.99	158.64

Table 2: Growth Rate for Genetically Modified Atlantic Salmon, *Salmo salar*.

The average weight¹⁷ of the salmon when released from the hatchery is 58.1 grams for a normal salmon and 232.3 grams for a genetically modified salmon.

- **Choose either genetically modified or normal salmon. Use Euler's method and the table provided below to calculate how much the salmon weigh 18 months after they have been released from the hatchery.**

Time since release (months)	Weight of salmon (grams)	Growth rate (grams per month)	Amount weight will change over next 3 months	New weight of salmon
0				
3				
6				
9				
12				
15				

¹⁷ Source: C. Clarke. (1998) "Seawater growth of Atlantic salmon, *Salmo salar*." *Aquaculture Update*, **81**: 1-2.

- Enter the growth rate data that you have used into your calculator and fit a curve to the data. Record your equation for growth rate here.

- The growth rate curves for normal and genetically modified are shown in Figures 8 and 9 below. Suppose that you were asked to approximate the area under the growth rate curve between $T = 0$ and $T = 18$ using 6 rectangles and your calculator. For whichever salmon you have been studying, sketch in the area that the calculator would actually find.

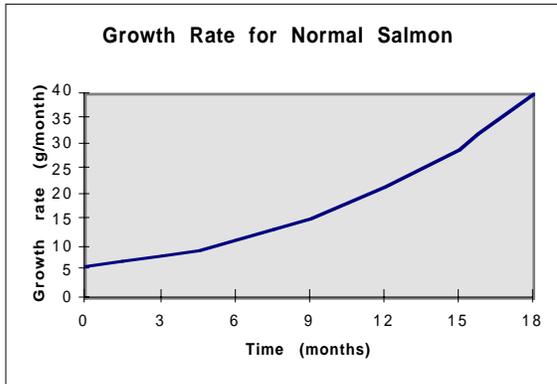


Figure 8: Graph showing growth rate versus time for normal Atlantic salmon, *Salmo salar*.

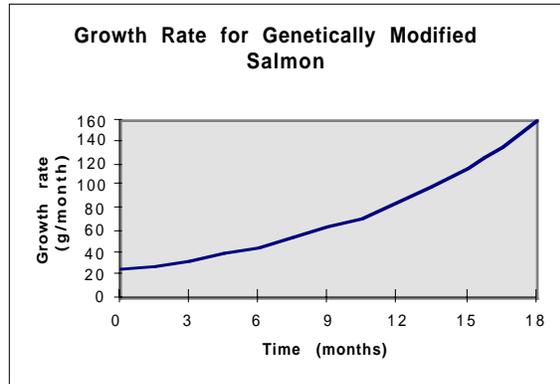


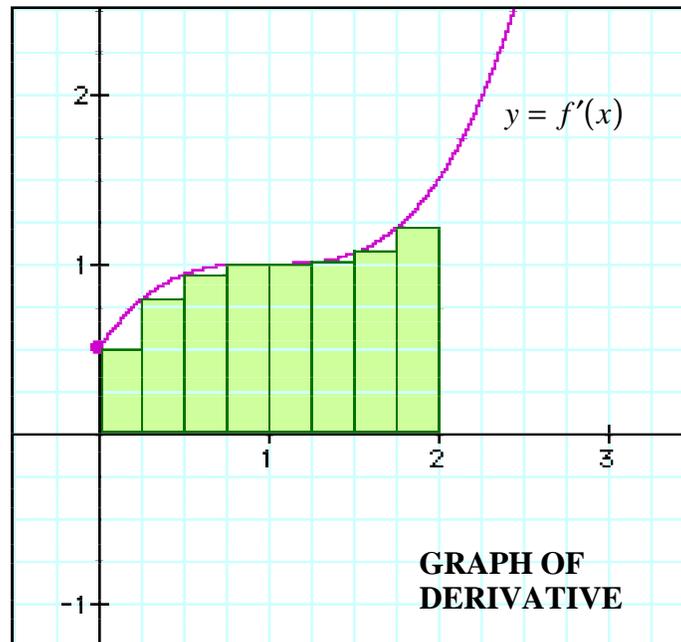
Figure 9: Graph showing growth rate versus time for genetically modified Atlantic salmon, *Salmo salar*.

- Use your equation and your calculator to find the area under the growth rate curve between $T = 0$ and $T = 18$ using 6 rectangles. Record the result in the table below. Then use as many rectangles as you think you need to get a really accurate figure for the area under the growth rate curve and record that area in the table below.

Result	Number of rectangles	Area the growth rate curve (from calculator)
Area that you sketched above	6	
Very accurate result for area under growth rate curve		

As you have calculated the amount that the salmon grow and the area under the growth rate curve you may have noticed some uncanny coincidences in the numbers that you come up with. In this very last part of the ICE you will set up an algebraic expression that we will discuss. The point of this discussion will be to connect Euler's method for approximating functions with area under a derivative graph.

- Consider the graph shown below. This is the graph of the derivative of a function.



Suppose that the width of each the rectangles is ΔT and that there are N rectangles in total. Explain why the total area of all of the rectangles will be given by:

$$\sum_{k=0}^{N-1} f'(k \cdot \Delta T) \cdot \Delta T.$$