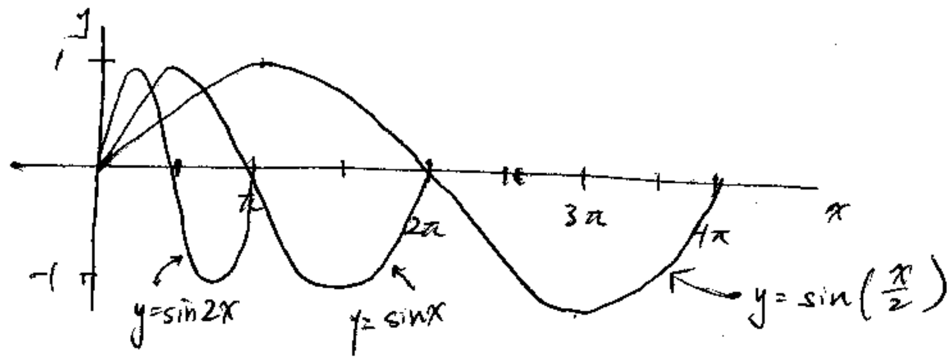
Assignment #12

3/5/2001

$\boxed{19.2}$

#2 a)



(only period of each function shown)

b) B determines the period of the function with:

$$\text{Period} = \frac{2\pi}{|B|}$$

#6 b) amplitude $= \dfrac{\frac{\pi}{4} - \left(-\frac{5\pi}{4}\right)}{2} = \dfrac{3\pi}{4}$

baseline @ $y = \frac{\pi}{4} - \frac{3\pi}{4} = -\frac{\pi}{2} \Longrightarrow$ vertical shift $= -\frac{\pi}{2}$

no horizontal shift if use $y = \cancel{\sin x}\ -\sin x$

Period = 4

Equation: $y = -\frac{3\pi}{4} \sin\left[\frac{\pi}{2}(x - 0)\right] - \frac{\pi}{2}$

$$\boxed{y = -\frac{3\pi}{4} \sin\left(\frac{\pi}{2} x\right) - \frac{\pi}{2}}$$

starts down amp determined by period vertical shift

c) baseline: $y = 1.5$ Period $= 6 = \frac{2\pi}{B}$; $B = \frac{\pi}{3}$

no horizontal shift if use $y = -\cos x$

amplitude $= 1.5$

Equation: $y = -1.5\cos\left(\frac{\pi}{3}x\right) + 1.5$

d) amplitude $= \frac{9-3}{2} = 3$ Period $= \frac{2}{3}(14-2) = 8$

baseline: $y = 6 \Rightarrow$ vertical shift $= +6$ up

use $y = -\sin x$

Equation: $y = -3\sin\left(\frac{\pi}{4}x\right) + 6$

#13 a) $y = x\sin x$

- amplitude of function ↑ as you move away from origin b/c x ↑.
- still oscillates across ~~y~~ x-axis b/c $\sin x$ oscillates
- symmetric across y-axis b/c the negative value of x on the left side of 0 reflects it across x-axis.

$\Rightarrow$ graph ii

b) $y = x^2 \sin x$

- amplitude increases VERY RAPIDLY as you move away from y-axis on both sides
- sign of function follows $\sin x$ b/c x^2 ALWAYS positive

$\Rightarrow$ graph iv.

c) $y = \sin(x^2)$

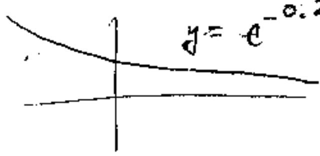
- symmetric across y-axis because $\sin(x^2)$ is same @ x and $-x$
- period shrinks as you move away from y-axis because

$$y = \sin(x^2) = \sin(x \cdot x)$$

this gets larger, so $\boxed{\frac{2\pi}{x}}$ gets smaller

period

$\Rightarrow$ graph v

d) 

so, $y = e^{-0.2x}\sin x$ has decreasing amplitude as you move in positive-x direction

$\Rightarrow$ graph iii

e) $y = e^{0.2x}\sin x$

- $e^{0.2x}$ is increasing function
- amp. of y increases in positive-x direction

$\Rightarrow$ graph i

f) $y = 0.5x + \sin x$

- $y = 0.5x$ is increasing linear function

$y = 0.5$ is negative in negative-x portion

$\Rightarrow$ graph vi

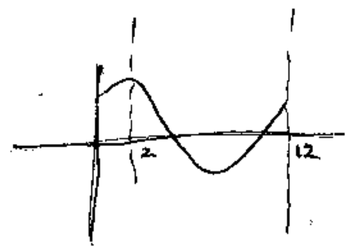
#14 a) amplitude $= \frac{450-230}{2} = 110$ mm

baseline $=$ 340 mm

peak @ $t = 2$ months

period = 1 year (12 months)

horiz. shift = 2



$$R(t) = 110\cos\left[\frac{\pi}{6}(t-2)\right] + 340$$

b) baseline value is average.

average = 340 mm/month

c) each year, predicts $\left(340\,\frac{\text{mm}}{\text{month}}\right)(12\text{ months}) = 4080$ mm

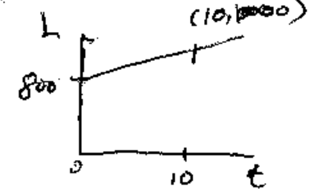
*smaller than recorded 4370 mm

% Error $= \frac{4370-4080}{4080} = 7.1\%$

of model

#18 a) i. known points: (0,800), (10,1000)

slope = 20   y-intercept = 800



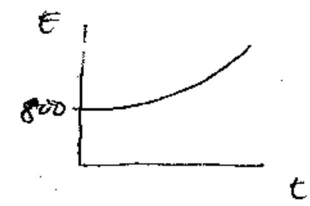
$L(t) = 20t + 800$

ii. $R = 1 +$ rate of increase %

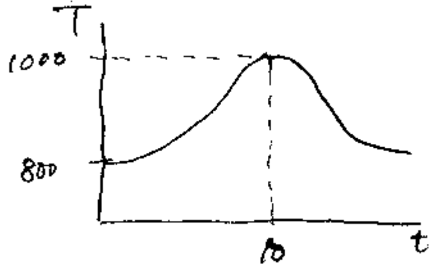
$800(R)^{10} = 1000$ (annual constant increase)

$R = 1.023$

$E(t) = 800(1.023)^t$



iii.



period = 20 $\Rightarrow B = \frac{\pi}{10}$

Amplitude = 100

Vertical Shift = + 900

$T(t) = -100\cos\left(\frac{\pi}{10}t\right) + 900$

b) in 2003, $t = 13$

$L(13) = 20(13) + 800 = \$1060$

$E(13) = 800(1.023)^{13} = \$1075.16$ ← (highest)

$T(13) = -100\cos\left[\left(\frac{\pi}{10}\right)(13)\right] + 900 = \$958.78$ (Lowest)

c) in 2020, $t = 30$

Alex: $L(30) = 20(30) + 800 = \$1400$

Jamey: $E(30) = 800(1.023)^{30} = \$1582.56$

Mike: $T(30) = -100\cos\left[\frac{\pi}{10}(30)\right] + 900 = \$1000$

d) i. exponential

ii. trigonometric

iii. linear

} look @ slopes of graphs