Assignment #14 3/9/2001

$\boxed{21.1}$

#2 $f(x) = \sin(x)$; estimate $f'(\pi)$

$$f'(\pi) \approx \frac{\sin(\pi + 0.0001) - \sin(\pi)}{0.0001} = \frac{\Delta y}{\Delta x}$$

$$= \boxed{-1}$$

$\boxed{21.2}$

#1 $$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x} = \frac{d}{dx}(\sin x)(\cos x)^{-1}$$

(use product rule)

$$\frac{d}{dx}(\sin x)(\cos x)^{-1} = (\sin x)(-1)(\cos^{-2} x)(-\sin x) + (\cos x)^{-1}(\cos x)$$

$$= \frac{\sin^2 x}{\cos^2 x} + \frac{\cos x}{\cos x} = 1 + \tan^2 x = \sec^2 x$$

pythagorean identity

#2 $$\frac{d}{dx}\sec x = \frac{d}{dx}(\cos x)^{-1} = (-1)(\cos^{-2} x)(-\sin x)$$

$$= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \sec x$$

#4 a) $y = \cos^2 x$

$$y' = 2\cos x\,(-\sin x) = -2\cos x \sin x$$

b) $y = \cos(x^2)$

$$y' = -\sin(x^2) \cdot (2x) = -2x\sin(x^2)$$

c) $y = x\tan^2 x$ (use product rule)

$$\frac{dy}{dx} = x(2\tan x)(\sec^2 x) + \tan^2 x \cdot (1)$$

$$= 2x\tan x \sec^2 x + \tan^2 x$$

d) $y = \sin^3(x^4)$

$$\frac{dy}{dx} = 3\sin^2(x^4) \cdot (\cos x^4)(4x^3)$$

$$= 12x^3 \sin^2(x^4)\cos(x^4)$$

e) (see last page of solution key)

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(5) a) $f(x) = e^{-0.3x}\sin x$ (set derivative $= 0$)

$$f'(x) = e^{-0.3x}\cos x + (\sin x)(e^{-0.3x})(-0.3) = 0$$

so, $\cancel{e^{-0.3x}}\cos x = +0.3\sin x\,(\cancel{e^{-0.3x}})$

$$\frac{1}{0.3} = \tan x \qquad x = 1.28$$

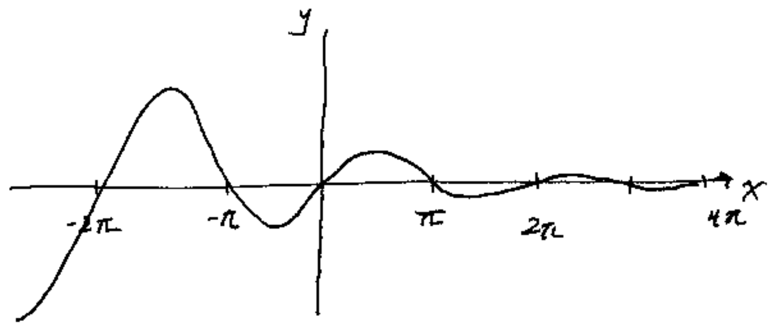
since 1.28 is between 0 and π, it must be a maximum (local)

local maxima: $1.28 + 2n\pi$, where n = integer

local minima: $(1.28 + \pi) + 2n\pi$, where n = integer

b) $f(x)$ is NOT periodic because the amplitude decreases as x increases. The function itself does not repeat.

c)



d) since amplitude decreases as x increases, the maximum value with $x \geq 0$ is the value at the first peak.

This occurs @ $x = 1.28$

max value = $\boxed{e^{-0.3(1.28)} \sin(1.28)}$

#6 a) $\frac{d}{dx} \sin(u(x)) = \cos(u(x)) \cdot u'(x)$

c) $\frac{d}{dx} u(x)(\sin x)$ (product rule)

$= \boxed{u(x)(\cos x) + (\sin x)\, u'(x)}$

4e) $y = 7\left[\cos(5x) + 3\right]^{x}$

take ln to bring x down

$$\ln y = x \ln\left\{7\left[\cos(5x)+3\right]\right\}$$

$$\ln y = x\left[\ln 7 + \ln\left[\cos(5x)+3\right]\right]$$

$$\ln y = x\ln 7 + x\ln\left[\cos(5x)+3\right]$$ ← take derivative

$$\frac{1}{y}\frac{dy}{dx} = \ln 7 + x\left(\frac{1}{\cos(5x)+3}\right)\left(-\sin(5x)\right)(5) + \ln\left[\cos(5x)+3\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \ln 7 - \frac{5x\sin(5x)}{\cos(5x)+3} + \ln\left[\cos(5x)+3\right]$$

$$\frac{dy}{dx} = \left[\ln 7 - \frac{5x\sin(5x)}{\cos(5x)+3} + \ln\left[\cos(5x)+3\right]\right]\cdot y$$

$$\frac{dy}{dx} = \left[\ln 7 - \frac{5x\sin(5x)}{\cos(5x)+3} + \ln\left[\cos(5x)+3\right]\right]\cdot 7\left[\cos(5x)+3\right]^{x}$$