Assignment #15

3/14/2001

$\boxed{21.2}$

#10 $\frac{d}{dx}\left[\frac{1}{\sin^3(\cos 2x)}\right] = \frac{d}{dx}\left[\sin^{-3}(\cos 2x)\right] = -3\sin^{-4}(\cos 2x)\cdot\cos(\cos 2x)\cdot(-\sin 2x)\cdot 2$

$$= \frac{6\left[\cos(\cos 2x)\right](\sin 2x)}{\sin^4(\cos 2x)}$$

#11 $\frac{d}{dx}\sqrt{\sin(2x^3)} = \frac{d}{dx}\sin^{\frac{1}{2}}(2x^3) = \frac{1}{2}\sin^{-\frac{1}{2}}(2x^3)\cdot\cos(2x^3)\cdot 6x^2$

$$= \frac{3x^2\cos(2x^3)}{\sqrt{\sin(2x^3)}}$$

#12 $\frac{d}{dx}\frac{4}{\sqrt{2-\cos(\frac{x}{7})}} = \frac{d}{dx}4\cdot\left(2-\cos(\tfrac{x}{7})\right)^{-\frac{1}{2}}$

$$= \left(-\tfrac{1}{2}\right)(4)\left(2-\cos(\tfrac{x}{7})\right)^{-\frac{3}{2}}\left(\sin(\tfrac{x}{7})\right)\cdot\tfrac{1}{7}$$

$$= -\frac{2}{7}\sin(\tfrac{x}{7})\cdot\left(2-\cos(\tfrac{x}{7})\right)^{-3/2}$$

#16 Using radians simplifies the derivatives of trigonometric functions.

$$\frac{d}{dx}\cos\left(\frac{\pi x}{180}\right) = -\sin\left(\frac{\pi x}{180}\right)\left(\frac{\pi}{180}\right)$$

no, derivative is not just $-\sin x°$

$\boxed{21.3}$

#1 a) find where $f'(x)=0$

$f'(x) = 1 + 2\cos x = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$x = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ for $0 \le x \le 2\pi$

critical points: $\frac{2\pi}{3} + 2n\pi$, $\frac{4\pi}{3} + 2n\pi$

b) if $f(x)$ increasing, $f'(x) > 0$.

$f'(x) = 1 + 2\cos x > 0$

$\cos x > -\frac{1}{2}$

<u>increasing</u> : $x \in [0, \frac{2\pi}{3}) \cup (\frac{4\pi}{3}, 2\pi]$

for $0 \le x \le 2\pi$

<u>decreasing</u> : $x \in (\frac{2\pi}{3}, \frac{4\pi}{3})$

or

$\frac{2\pi}{3} < x < \frac{4\pi}{3}$ for $0 \le x \le 2\pi$

c) use critical points $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$

use $f''(x)$ to determine concavity

$f'(x) = 1 + 2\cos x$

$f''(x) = -2\sin x$

$f''(\frac{2\pi}{3}) = -2\sin(\frac{2\pi}{3}) = -2(\frac{\sqrt{3}}{2}) = -\sqrt{3} < 0$

@ $x = \frac{2\pi}{3} + 2n\pi$, concave down. <u>Maximum</u>

$f''(\frac{4\pi}{3}) = -2\sin(\frac{4\pi}{3}) = -2(-\frac{\sqrt{3}}{2}) = \sqrt{3} > 0$

@ $x = \frac{4\pi}{3} + 2n\pi$, concave up. <u>minimum</u>

d) if domain unrestricted, <u>no</u> absolute minima or maxima

– b/c x can get infinitely big or small

e) $f''(x) = -2\sin x$

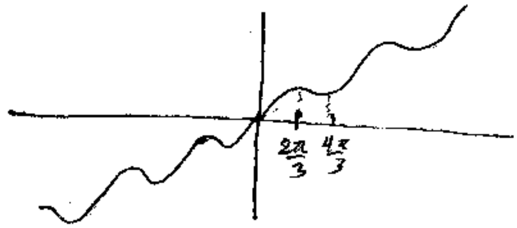
<u>concave up</u> : $-2\sin x > 0 \Rightarrow \sin x < 0$ (divide by -2)

$\boxed{\pi < x < 2\pi}$ for $0 < x < 2\pi$

<u>concave down</u> : $-2\sin x < 0 \Rightarrow \sin x > 0$

$\boxed{0 < x < \pi}$ for $0 < x < 2\pi$

f) $f(x) = x + 2\sin x$



#7 a) we know $\sin(0) = 0$

$$\text{slope of } \sin(x) = \cos(x)$$
$$\text{slope @ } 0 = \cos(0) = 1$$

tangent line approx:

$$\sin(0.2) = (\text{slope})(\Delta x)$$
$$= (1)(0.2) = \boxed{0.2}$$

d) $\sin(0) = 0$ slope @ $x=0$, is 1

$$\sin(-0.1) = (\text{slope})(\Delta x) = (1)(-0.1)$$
$$= \boxed{-0.1}$$

#12 a) $\text{Distance} = D = \dfrac{V_0^2 \sin(2\theta)}{g}$

$$\frac{dD}{d\theta} = \frac{V_0^2}{g}\cos(2\theta)\cdot 2 = \frac{2V_0^2\cos(2\theta)}{g} = 0$$

$\cos(2\theta) = 0$ (assume V_0 is constant)

$2\theta = \frac{\pi}{2}$ (we're only concerned about $0 < x < \pi/2$)

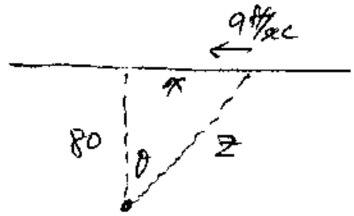
$$\boxed{\theta = \frac{\pi}{4}}$$

$$\text{Distance}_{max} = \frac{V_0^2 \sin\left(2\cdot\frac{\pi}{4}\right)}{g} = \boxed{\frac{V_0^2}{g}}$$

b) $100\,\frac{mi}{hr} \times \frac{5280\,ft}{1\,mi} \times \frac{1\,m}{3.28\,ft} \times \frac{1\,hr}{3600\,sec} = 44.7\ m/s$

$$D = \frac{V_0^2}{g} = \frac{(44.7\ m/s)^2}{9.8\ m/s^2} = \boxed{204\ m}$$

#13



we know $\frac{dx}{dt} = -9$

we want $\frac{d\theta}{dt}$

we need an equation with both θ and x

$$\tan\theta = \frac{x}{80}$$

$$\frac{d}{dt}\tan\theta = \frac{d}{dt}\frac{x}{80}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{80}\frac{dx}{dt}$$

$$\sec\theta = \frac{hyp}{adj} = \frac{z}{80} = \frac{100}{80}$$

$$\left(\frac{100}{80}\right)^2 \frac{d\theta}{dt} = \frac{1}{80}(-9)$$

$$\frac{d\theta}{dt} = -0.072 \text{ rad/sec}$$

(negative b/c θ decreasing)