Assignment #17

3/19/2001

21.4

#1 $y = 3\tan x - 4\tan^{-1}x$

$$\frac{dy}{dx} = 3\sec^2 x - 4\left(\frac{1}{1+x^2}\right)$$

#2 $y = f(x) = 3\tan^{-1}(2\sqrt{x})$

$$f'(x) = 3\left(\frac{1}{1+(2\sqrt{x})^2}\right)\cdot\left(x^{-\frac{1}{2}}\right)$$

$$= \frac{3}{1+4x}\cdot\frac{1}{\sqrt{x}}$$

#3 $y = \sin x \cdot \arcsin x$

$$\frac{dy}{dx} = \sin x\left(\frac{1}{\sqrt{1-x^2}}\right) + \arcsin x\,(\cos x)$$

#4 $y = \sqrt{\tan^{-1}x} = (\tan^{-1}x)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(\tan^{-1}x)^{-\frac{1}{2}}\cdot\frac{1}{1+x^2}$$

$$= \frac{1}{2\sqrt{\tan^{-1}x}}\cdot\frac{1}{1+x^2}$$

#7 a) $y = \cos^{-1}x$

$\cos y = x$

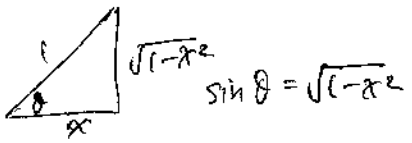
$\frac{d}{dx}\cos y = \frac{d}{dx}x$

$-\sin y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = -\frac{1}{\sin y}$ $\qquad y = \cos^{-1}(x)$

$\frac{dy}{dx} = -\frac{1}{\sin(\cos^{-1}x)}$

$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$



$\sin\theta = \sqrt{1-x^2}$

b) domain: $-1 \le x \le 1$

c) range: $0 \le y \le \pi$

d) decreasing throughout entire domain

e) find 2nd derivative

$$\text{derivative} = \frac{-1}{\sqrt{1-x^2}}$$

$$\text{2nd deriv} = \frac{1}{2}(1-x^2)^{-\frac{3}{2}} \cdot (-2x)$$

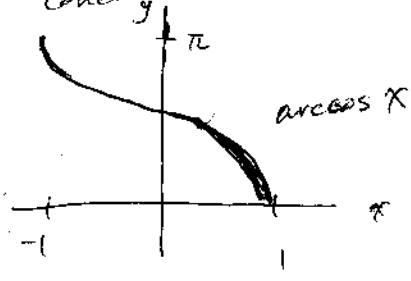
$$= \frac{-x}{(1-x^2)^{+3/2}}$$ ← denominator always positive

inflection point @ $x = 0$

concave up: $[-1, 0)$

concave down: $(0, 1]$

f)



(#9) $$\frac{d}{dx}\frac{\sin^{-1}x}{\cos^{-1}x} = \frac{(\cos^{-1}x)\left(\frac{1}{\sqrt{1-x^2}}\right) - (\sin^{-1}x)\left(\frac{-1}{\sqrt{1-x^2}}\right)}{(\cos^{-1}x)^2}$$

this does not equal $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$