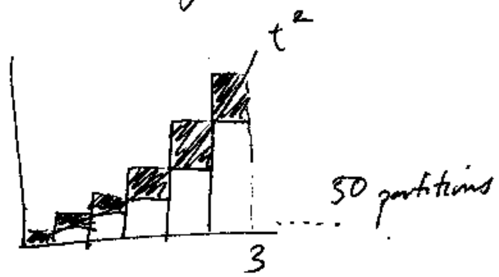
Assignment # 19

04/02/2001

$\boxed{22.1}$

#8 d)



50:

$$R_{50} - L_{50} = \underbrace{\left(\frac{3}{50}\right)}_{\text{width of each rectangle}} \underbrace{\left(v(3) - v(0)\right)}_{\text{sum of heights of shaded rectangles}}$$

$$= \frac{3}{50}\left(3^2\right) = \boxed{\frac{27}{50}}$$

100:

$$\left(\frac{3}{100}\right)\left(v(3) - v(0)\right)$$

$$= \frac{3}{100}\left(9\right) = \boxed{\frac{27}{100}}$$

e)

$$\frac{3}{n}\left(9\right) \leq 0.01$$

$$\frac{3}{n} \leq \frac{0.01}{9}$$

$$\frac{3}{\frac{0.01}{9}} \leq n$$

$$\boxed{2700 \leq n}$$

#9 upper bound

sum of areas is larger than actual area under the function!

22.2

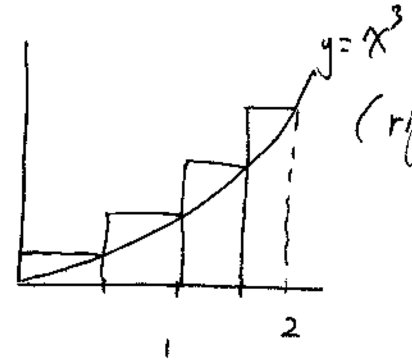
#1 distances traveled

A) $\int_0^5 f(t)\,dt$ ⇐ greatest

B) $\int_0^5 g(t)\,dt$

C) $\int_0^5 h(t)\,dt$ ⇐ smallest

#4 a)



(right-hand Riemann sum pictured)

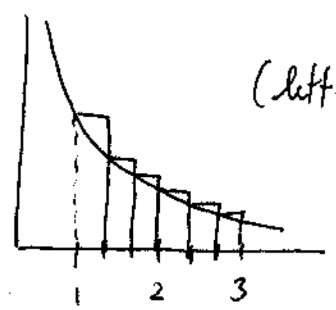
<u>upper bound</u> (right handed)

$$0.5\left(0.5^3 + 1^3 + 1.5^3 + 2^3\right) = 6.25$$

<u>lower bound</u> (left-handed)

$$0.5\left(0^3 + 0.5^3 + 1^3 + 1.5^3\right) = 2.25$$

b)



(left-hand Riemann sum pictured)

<u>upper</u> (left-handed approx)

$$\frac{1}{3}\left(1 + \frac{3}{4} + \frac{3}{5} + \frac{3}{6} + \frac{3}{7} + \frac{3}{8}\right) = 1.22$$

<u>lower</u> (right-handed)

$$\frac{1}{3}\left(\frac{3}{4} + \frac{3}{5} + \frac{3}{6} + \frac{3}{7} + \frac{3}{8} + \frac{3}{9}\right) \approx 1.00$$

(#5) (be sure to use special figure handed out)

a) $\int_0^1 V_c(t)\,dt$; area under $V_c(t)$ from 0 to 1

b) $\underbrace{\int_0^{1.5} V_c(t)\,dt}_{\text{distance chicken travelled}} - \underbrace{\int_0^{1.5} V_g(t)\,dt}_{\text{distance goat travelled}}$

c) $\left| \int_0^3 V_c(t)\,dt - \int_0^3 V_g(t)\,dt \right| =$ distance

d) $t = 6.5$

the distance between the animals keeps increasing as long as $\underline{V_g(t) > V_c(t)}$

e) $t \approx 3$ and $t \approx 8.5$

f) $t \approx 3.5$

- rate of increase of distance between animals $= V_g - V_c$
- to maximize, take derivative and set equal to 0

$$(V_g - V_c)' = 0$$
$$\underline{V_g' - V_c' = 0}$$

(#6) a) ascending: $L_4, L_{20}, L_{100}, \int_0^2 f(x)\,dx, R_{100}, R_{20}, R_4$

b) $|R_4 - L_4| = \frac{2}{4}\left(f(2) - f(0)\right) = \frac{1}{2}(8) = 4$

c) $|R_{100} - L_{100}| = \frac{2}{100}\left(f(2) - f(0)\right) = \frac{2}{100}(8) = 0.16$

d) $|R_n - L_n| = \frac{2}{n}(f(2) - f(0))$

$$\frac{2}{n}(8) < 0.05$$

$$\frac{2}{n} < \frac{0.05}{8}$$

$$\frac{2}{\frac{0.05}{8}} < n \implies \boxed{320 < n}$$

e) $R_4 = \frac{2}{4}(0.5^3 + 1^3 + 1.5^3 + 2^3)$

$$= \frac{2}{4}\sum_{k=1}^{4}\left[(0.5)k\right]^3 = \boxed{12.5}$$

#7 a) $\underbrace{\int_9^{10} r(t)\,dt}_{\text{ppl that came in}} - 15$ (line starts @ $t = 9$)

↑ ppl served

b) $\int_9^{15} r(t)\,dt - 15(6)$

c) $\int_8^{16} r(t)\,dt$

d) $\int_8^{16} r(t)\,dt$; assume everyone eventually served

e) $\int_9^{16} r(t)\,dt - 7(15)$

f) $\frac{\int_9^{12} r(t)\,dt - 3(15)}{15} = \frac{\text{people in line}}{\text{rate of service}}$

g) $\frac{\int_9^{16} r(t)\,dt - 7(15)}{15} = \frac{\text{length of line @ } t = 4}{\text{rate of service}}$