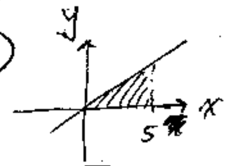
Assignment #20

4/4/2001

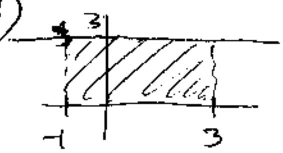
$\boxed{22.3}$

#1 a) 

Area $= \frac{1}{2}(5\times5) = \boxed{12.5}$

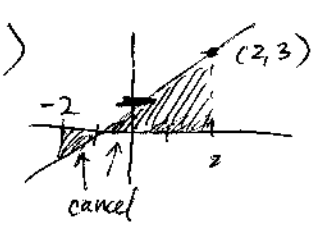
b) $\boxed{0}$ (areas cancel)

c) $2\left(\frac{1}{2}(2\times2)\right) = \boxed{4}$

d) 

$4\times3 = \boxed{12}$

e) $\boxed{0}$; areas cancel

g) 

Area $= \frac{1}{2}(3\times3) - \frac{1}{2}(1\times1)$

$= \frac{1}{2}(9-1) = \boxed{4}$

#2 $\boxed{f}$

- we know $\int_{-6}^{3} f(x)dx$ is positive because the function is positive throughout the entire interval
- estimate area under curve ≈ 9

#3 $\int_{-\pi}^{2\pi} \sin(t)\,dt < \int_0^{\pi} \cos(2t)\,dt = \int_{-\pi}^{2\pi} \cos(t)\,dt <$

$\int_0^{\pi/2} \cos(t)\,dt < \int_0^{\pi} \sin(t)\,dt < \int_0^{3\pi/2} |\sin(t)|\,dt$

#6 a) $\int_{-2}^{3} f(x)dx = \frac{1}{2}(2\times4) + \frac{1}{4}(\pi 3^2) = \boxed{4+\frac{9}{4}\pi}$

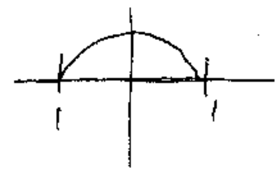
b) $\int_{-8}^{6} f(x)dx = \frac{1}{2}(6\times4) + \frac{1}{2}(2\times4) + \frac{1}{2}(\pi 3^2) = 16 + \frac{9}{2}\pi$

c) $\int_0^6 |f(x)|dx = \frac{1}{2}(\pi 3^2) = \frac{9}{2}\pi$

$\boxed{22.4}$

#2 a) $y = \sqrt{1-x^2}$ is the function

$\Rightarrow x^2 + y^2 = 1 \longrightarrow$



top part of circle

$$\int_0^1 \sqrt{1-x^2}\,dx = \frac{1}{4}(\pi 1^2) = \boxed{\frac{\pi}{4}}$$

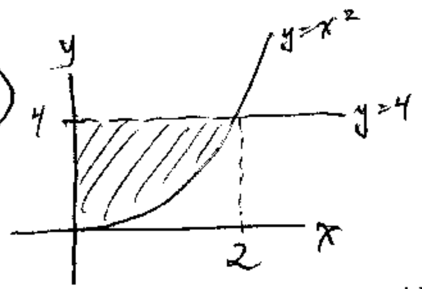
#5 a) $\int_a^b |f(t)|\,dt \geq \left| \int_a^b f(t)\,dt \right|$

the entire function is positive. Add all areas

the right-side function could cancel itself.

b) if the function $f(t)$ is always on the same side of the ~~as~~ x-axis $\Rightarrow$ always positive OR always negative

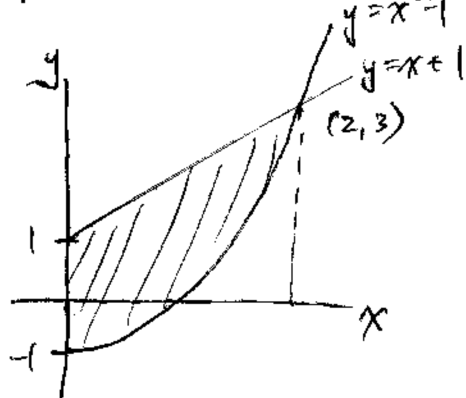
#6 a)



$$\int_0^2 4\,dx - \int_0^2 x^2\,dx$$

$$= \int_0^2 4 - x^2\,dx$$

b)



$x + 1 = x^2 - 1$

$2 = x^2 - x$ $\cancel{x(x-1)}$

~~one solution (1,2)~~

$0 = x^2 - x - 2$

$= (x+1)(x-2)$

one solution: $(2,3)$

$$\boxed{\int_0^2 x+1\,dx - \int_0^2 x^2-1}$$