# Assignment #23

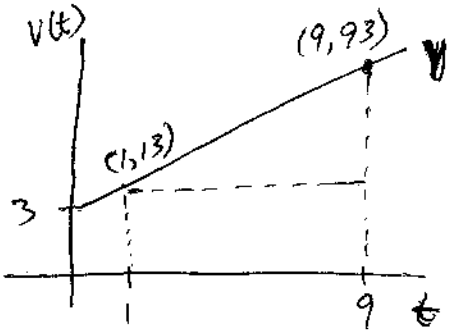
4/18/2001

24.1

(#1) b) area under one arc $= \int_{-\pi/2}^{\pi/2} \cos x\,dx = \sin x \Big|_{x=-\pi/2}^{x=\pi/2}$

$= \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) = 1 - (-1) = \boxed{2}$

(#2)




Geometrically,

Area $= (9-1)(13) + \frac{(8 \cdot 80)}{2}$

$= \boxed{424}$

$\int_1^9 (10t+3)\,dt = \left[5t^2+3t\right]_1^9 = (5\cdot 9^2 + 3\cdot 9) - (5+3)$

$= \boxed{424}$

(#3) $\int_1^2 t^3\,dt = \frac{1}{4}t^4 \Big|_{t=1}^{t=2} = \frac{1}{4}(2)^4 - \frac{1}{4}(1)^4$

$= \frac{16}{4} - \frac{1}{4} = \boxed{3\frac{3}{4}}$

(#4) $\int_1^3 \frac{1}{t}\,dt = \ln t \Big|_1^3 = \ln 3 - \ln 1 = \boxed{\ln 3}$

(#5) area $= \int_0^1 e^x\,dx = e^x \Big|_0^1 = e^1 - e^0 = \boxed{e-1}$

(#7) rate $= 1.5t + \sqrt{t}$

#hearts $= \int_0^9 1.5t + \sqrt{t}\,dt = \int_0^9 1.5t + t^{1/2}\,dt = \left[\frac{3}{4}t^2 + \frac{2}{3}t^{3/2}\right]_0^9$

$= \frac{3}{4}(81) + \frac{2}{3}(27) = \boxed{78.75}$

(#8) a) $-2e^{-t} = -\frac{2}{e^t}$

this is always a negative number.

∴ it is cooling.

b) $\int_0^1 -2e^{-t}\,dt = 2e^{-t}\Big|_0^1 = \boxed{\frac{2}{e} - 2}$

c) $\int_1^2 -2e^{-t}\,dt = 2e^{-t}\Big|_1^2 = \boxed{\frac{2}{e^2} - \frac{2}{e}}$

d) how hot @ $t = T$ ?

amount of change $= \int_0^T -2e^{-t}\,dt$

temp @ $t = T$ is $\boxed{100 - \int_0^T -2e^{-t}\,dt}$

(#9) a) $f(1) = \int_1^1 \frac{1}{t}\,dt = 0$

$f(5) = \int_1^5 \frac{1}{t}\,dt = \ln t\Big|_1^5 = \ln 5 - \ln 1 = \boxed{\ln 5}$

$f(10) = \int_1^{10} \frac{1}{t}\,dt = \ln t\Big|_1^{10} = \boxed{\ln 10}$

$f(\frac{1}{2}) = \int_1^{1/2} \frac{1}{t}\,dt = \ln t\Big|_1^{1/2} = \ln \frac{1}{2} - \ln 1 = \boxed{\ln \frac{1}{2}}$

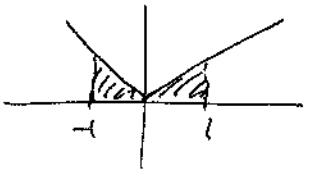
b) alternative formula

$f(x) = \ln x$

c) $\ln x = \int_1^x \frac{1}{t}\,dt$

as $x$ increases, the area under the function $f(t) = \frac{1}{t}$ increases, but at a decreasing rate. Picture moving in increments to the right on $f(t) = \frac{1}{t} \Rightarrow$ You add area to the integral, but in smaller and smaller increments

#14 a)




$$\int_{-1}^{1} |x|\,dx = 2\left(\frac{1\cdot 1}{2}\right) = 1$$

b) $\left.\frac{|x^2|}{2}\right|_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = \boxed{0}$ ✗

this does not equal part a.

c) on $[-1, 0)$, antiderivative $= -\frac{1}{2}x^2$

on $(0, 1]$, antiderivative $= \frac{1}{2}x^2$