Assignment #25 4/16/2001

25.1

#18 a) $\int \frac{2+x}{x}dx = \int \frac{2}{x}+1\,dx = \boxed{2\ln x + x + c}$

b) $\int \frac{3}{x^2}dx = 3\int x^{-2}dx = \boxed{-3x^{-1}+c}$

c) $\int \frac{3}{1+x^2}dx = 3\int \frac{1}{1+x^2}dx = \boxed{3\tan^{-1}x + c}$

d) $\int \left(\frac{t^3}{4}+\frac{4}{\sqrt{t}}\right)dt = \int \frac{1}{4}t^3\,dt + \int 4t^{-1/2}dt = \frac{1}{16}t^4 + 4(2t^{1/2}) + c$

$= \frac{t^4}{16} + 8\sqrt{t} + c$

#19 a) $\int (x+1)\sqrt{5x}\,dx = \sqrt{5}\int \sqrt{x}(1+x)\,dx = \sqrt{5}\int (\sqrt{x} + x^{3/2})dx$

$\cancel{= \sqrt{5}\left(\frac{2}{3}x^{3/2} + \frac{1}{2}x^2\right) + c} = \sqrt{5}\left(\frac{2}{3}x^{3/2} + \frac{2}{5}x^{5/2}\right) + c$

25.2

#1 a) $\int \frac{1}{x}dx = \ln x + c$

b) $\int \frac{1}{x+1}dx \quad \begin{matrix} u = x+1 \\ du = dx \end{matrix} \Rightarrow \int \frac{1}{u}du = \ln u + c$

$= \ln|x+1| + c$

c) $\int \frac{1}{(x+1)^2} \quad \begin{matrix} u = x+1 \\ du = dx \end{matrix} \Rightarrow \int \frac{1}{u^2}du = -u^{-1} + c = \boxed{-(x+1)^{-1} + c}$

d) $\int \frac{1}{x^2+1}dx = \tan^{-1}x + c$

e) $\int \frac{x}{x^2+1}dx \quad \begin{matrix} u = 1+x^2 \\ du = 2x\,dx \end{matrix} \Rightarrow \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u + c = \frac{1}{2}\ln|1+x^2| + c$

f) $\int \frac{x^2+1}{x}dx = \int x + \frac{1}{x}dx = \frac{1}{2}x^2 + \ln x + c$

g) $\int (1+x)^5 dx \quad \begin{matrix} u = 1+x \\ du = dx \end{matrix} = \int u^5 du = \frac{1}{6}u^6 + c = \frac{1}{6}(1+x)^6 + c$

h) $\int \frac{1}{(1+x)^5}dx \quad \begin{matrix} u = 1+x \\ du = dx \end{matrix} = \int \frac{1}{u^5}du = -\frac{1}{4}u^{-4} + c = -\frac{1}{4}(1+x)^{-4} + c$

i) $\int (1+x^2)^2 dx = \int 1 + 2x^2 + x^4 dx = x + \frac{2}{3}x^3 + \frac{1}{5}x^5 + c$

#2 a) $\int 3\sin(5t)\,dt$ $\quad u=5t,\ du=5\,dt \Rightarrow \frac{1}{5}\int 3\sin u\,du = -\frac{3}{5}\cos u + C$

$= \boxed{-\frac{3}{5}\cos(5t)+C}$

b) $\int \pi\cos(\pi t)\,dt$ $\quad u=\pi t,\ du=\pi\,dt \Rightarrow \int \cos u\,du = \sin u + C$

$= \boxed{\sin(\pi t)+C}$

c) $\int \sqrt{3x+5}\,dx$ $\quad u=3x+5,\ du=3\,dx \Rightarrow \frac{1}{3}\int \sqrt{u}\,du = \frac{1}{3}\left(\frac{2}{3}u^{3/2}\right)+C$

$= \boxed{\frac{2}{9}(3x+5)^{3/2}+C}$

d) $\int \frac{\pi}{e^x}\,dx = \pi\int e^{-x}\,dx$ $\quad u=-x,\ du=-dx \Rightarrow -\pi\int e^u\,du$

$= -\pi e^u = \boxed{-\pi e^{-x}+C}$

e) $\int e^{-3t}\,dt$ $\quad u=-3t,\ du=-3\,dt \Rightarrow -\frac{1}{3}\int e^u\,du = -\frac{1}{3}e^u + C = \boxed{-\frac{1}{3}(e^{-3t})+C}$

f) $\int \sqrt{e^t}\,dt = \int e^{t/2}\,dt$ $\quad u=\frac{t}{2},\ du=\frac{1}{2}dt \Rightarrow 2\int e^u\,du = 2e^u + C = \boxed{2e^{t/2}+C}$

g) $\int \frac{6}{\sqrt{t^3}}\,dt = 6\int t^{-3/2}\,dt = 6\left(-2t^{-1/2}\right)+C = \boxed{-12\frac{1}{\sqrt{t}}+C}$

h) $\int \frac{1}{3t+8}\,dt$ $\quad u=3t+8,\ du=3\,dt \Rightarrow \frac{1}{3}\int \frac{1}{u}\,du = \boxed{\frac{1}{3}\ln|3t+8|+C}$

#4 a) $\int x\sqrt{2x^2+1}\,dx$ $\quad u=2x^2+1,\ du=4x\,dx \Rightarrow \frac{1}{4}\int \sqrt{u}\,du = \frac{1}{4}\left(\frac{2}{3}u^{3/2}\right)+C$

$= \boxed{\frac{1}{6}(2x^2+1)^{3/2}+C}$

b) $\int \frac{x}{\sqrt{2x^2+1}}\,dx$ $\quad u=2x^2+1,\ du=4x\,dx \Rightarrow \frac{1}{4}\int \frac{du}{\sqrt{u}} = \frac{1}{4}\left(2u^{1/2}\right)+C$

$= \boxed{\frac{1}{2}(2x^2+1)^{1/2}+C}$

c) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ $\quad u=\sqrt{x},\ du=\frac{1}{2\sqrt{x}}dx \Rightarrow 2\int \cos u\,du = \boxed{2\sin\sqrt{x}+C}$

d) $\int \sqrt{\cos x}\,\sin x\,dx$ $\quad u=\cos x,\ du=-\sin x\,dx \Rightarrow -\int \sqrt{u}\,du = -\frac{2}{3}u^{3/2}+C$

$= \boxed{-\frac{2}{3}(\cos x)^{3/2}+C}$

#5 a) $\int \frac{5}{1+9x^2}\,dx$ $\quad u=3x,\ du=3dx$ $\Rightarrow \frac{5}{3}\int \frac{1}{1+u^2}\,du = \frac{5}{3}\tan^{-1}u + C$

$= \boxed{\frac{5}{3}\tan^{-1}(3x)+C}$

b) $\int \frac{\ln x}{x}\,dx$ $\quad u=\ln x,\ du=\frac{1}{x}dx$ $\Rightarrow \int u\,du = \frac{1}{2}u^2+C = \boxed{\frac{1}{2}(\ln x)^2+C}$

c) $\int \frac{e^x}{e^{-x}}\,dx = \int e^{2x}dx$ $\quad u=2x,\ du=2dx$ $\Rightarrow \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u+C$

$= \boxed{\frac{1}{2}e^{2x}+C}$

d) $\int \frac{(\ln w)^2}{w}\,dw$ $\quad u=\ln w,\ du=\frac{1}{w}dw$ $\Rightarrow \int u^2\,du = \frac{1}{3}u^3+C$

$= \boxed{\frac{1}{3}(\ln w)^3+C}$

#15 $\int_0^{\pi/4} \tan x\,dx = \int_0^{\pi/4} \frac{\sin x}{\cos x}\,dx$ $\quad u=\cos x,\ du=-\sin x\,dx$

$\Rightarrow -\int_0^{\pi/4} \frac{du}{u} = -\ln|u|\Big|_0^{\pi/4} = -\ln(\cos x)\Big|_0^{\pi/4}$

$= -\ln\left(\frac{\sqrt{2}}{2}\right) - \left[-\ln(1)\right] = \boxed{-\ln\left(\frac{\sqrt{2}}{2}\right)}$