Assignment #26

4/20/2001

25.2

#22 $\int \sec^2 x \tan^2 x \, dx$

$u = \tan x$
$du = \sec^2 x \, dx$ $\Rightarrow \int u^2 du = \frac{1}{3}u^3 + C$

$= \boxed{\frac{1}{3}(\tan x)^3 + C}$

#23 $\int \frac{\sec^2\sqrt{x}\,\tan^2\sqrt{x}}{\sqrt{x}}\,dx$

$u = \tan\sqrt{x}$
$du = \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}}dx \Rightarrow 2\int u^2 du = 2\left(\frac{1}{3}u^3\right) + C$

$= \boxed{\frac{2}{3}\tan^3\sqrt{x} + C}$

~~25.3~~

#25 a) $\int \cos x \sin x \, dx$ $\quad u = \sin x$, $du = \cos x$ $\Rightarrow \int u \, du$

$= \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2 x + C$

b) $\int \cos x \sin x \, dx$ $\quad u = \cos x$, $du = -\sin x$ $\Rightarrow -\int u \, du$

$= -\frac{1}{2}u^2 + C = -\frac{1}{2}\cos^2 x + C$

c) to show they're the same, use $1 = \sin^2 x + \cos^2 x$

$\frac{1}{2}\sin^2 x + C = \frac{1}{2}(1 - \cos^2 x) + C = \frac{1}{2} - \frac{1}{2}\cos^2 x + C$

$= -\frac{1}{2}\cos^2 x + \frac{1}{2} + C = -\frac{1}{2}\cos^2 x + C$

new C

$\boxed{25.3}$

#3 $\int x\sqrt{3x+5}\,dx$ $\quad u = 3x+5$, $du = 3dx$ $\to$ $x = \frac{u-5}{3}$

$= \frac{1}{3}\int\left(\frac{u-5}{3}\right)\sqrt{u}\,du = \frac{1}{9}\int(u-5)\sqrt{u}\,du = \frac{1}{9}\int u^{3/2} - 5u^{1/2}\,du$

$= \frac{1}{9}\left(\frac{2}{5}u^{\frac{5}{2}} - 5\left(\frac{2}{3}\right)u^{3/2}\right) + C = \boxed{\frac{1}{9}\left(\frac{2}{5}u^{\frac{5}{2}} - \frac{10}{3}u^{3/2}\right) + C}$

#4 $\int \frac{2x}{3+x}$ $\quad u = 3+x$, $du = dx$ $\to$ $x = u-3$

$= \int \frac{2(u-3)}{u}du = 2\int \frac{u-3}{u}du = 2\int \frac{u}{u} - \frac{3}{u}\,du = 2\int 1 - \frac{3}{u}\,du$

$= 2(u - 3\ln|u|) + C = 2(3+x) - 6\ln|x+3| + C$

$= \boxed{6 + 2x - 6\ln|x+3| + C}$

#5 $\int 3t\sqrt{t^2+5}\,dt$ (like last week's hwk)

$u = t^2+5$
$du = 2t\,dt$

$\int 3t\sqrt{t^2+5}\,dt = \frac{3}{2}\int \sqrt{u}\,du = \frac{3}{2}\left(\frac{2}{3}u^{3/2}\right) + C$

$= u^{3/2} + C = (t^2+5)^{3/2} + C$

#7 $\int \frac{x^\pi}{5}dx = \frac{1}{5}\int x^\pi dx = \boxed{\frac{1}{5}\left(\frac{1}{\pi+1}\right)x^{\pi+1} + C}$

#12 $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}dx$ $\quad$ use $x = \sin\theta$, $dx = \cos\theta\,d\theta$

$= \int_{x=0}^{x=1/2} \frac{1}{\sqrt{1-\sin^2\theta}}(\cos\theta\,d\theta) = \int_{x=0}^{x=1/2}\frac{\cos\theta}{\sqrt{\cos^2\theta}}d\theta = \int_{x=0}^{x=\frac{1}{2}} d\theta$

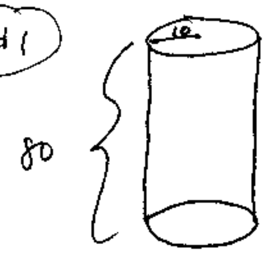
$= \theta\Big|_{x=0}^{x=\frac{1}{2}}$ $\left(\theta = \sin^{-1}x, \text{ since } x = \sin\theta\right)$

$$\text{so, } \theta \Big|_{x=0}^{x=\frac{1}{2}} = \sin^{-1} x \Big|_{x=0}^{x=\frac{1}{2}} = \sin^{-1}\left(\tfrac{1}{2}\right) - \sin^{-1}(0)$$

$$= \frac{\pi}{6} - 0 = \boxed{\frac{\pi}{6}}$$

$\boxed{27.1}$

#1



density @ each height $h = \rho(h)$
$\therefore$ to get total mass, add the mass of each infinitesimally thin slice from bottom to top.

$$(\text{mass of ea. slice}) = \rho(h) \cdot (\text{Volume of each slice})$$

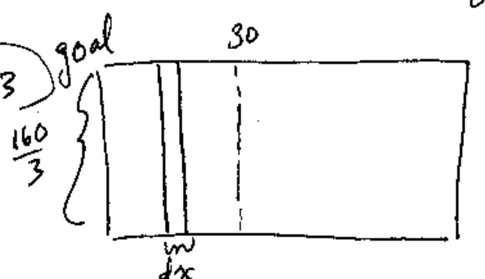
$$= \rho(h) \cdot \pi r^2 \, dh$$

infinitesimally thin

$$\text{mass} = \boxed{\int_0^{80} \rho(h) \cdot \pi (10)^2 \, dh}$$

$$\text{or} \quad 100\pi \int_0^{80} \rho(h)\, dh$$

#3



density @ each distance $= \frac{30-x}{3}$
• divide field into stripes which have a given orange density

$$(\text{oranges in ea. slice}) = \rho(x) \cdot (\text{area of each slice})$$

$$= \rho(x) \cdot \left(\frac{160}{3}\right) dx$$

$$\text{total oranges}: \boxed{\int_0^{30} \left(\frac{30-x}{3}\right)\left(\frac{160}{3}\right) dx}$$