Assignment #27 4/23/2001

[25.3]

#13 $\int \frac{2}{x(x+2)}\,dx = \int \frac{A}{x} + \frac{B}{x+2}\,dx$

so, $2 = A(x+2) + Bx$ (numerators equal if denominator equal)

$2 = Ax + 2A + Bx$

so, $2 = 2A$ and $0 = Ax + Bx$

$\boxed{A = 1}$ $-B = A$

$\boxed{B = -1}$

so, integral becomes:

$\int \frac{1}{x} + \frac{-1}{x+2}\,dx = \ln|x| + (-1)\ln|x+2| + C$

$= \boxed{\ln|x| - \ln|x+2| + C}$

#14 $\int \frac{1}{x^2-1}\,dx = \int \frac{1}{(x+1)(x-1)}\,dx = \int \frac{A}{x+1} + \frac{B}{x-1}\,dx$

so, set numerators equal: $1 = A(x-1) + B(x+1)$

$1 = Ax - A + Bx + B$

so, $0 = Ax + Bx$ and $1 = -A + B$

$0 = A + B$

$-)\ 1 = -A + B$

$-1 = 2A + 0B$

$\boxed{A = -\frac{1}{2}}$

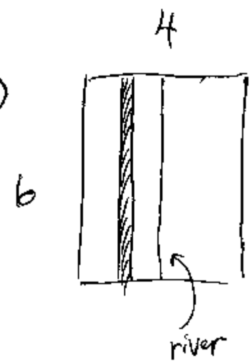
$\boxed{B = \frac{1}{2}}$

integral becomes: $\int \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}\,dx$

$= \boxed{-\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C}$

27.1

(#2) a)



- we need to slice into vertical stripes as shown. we can find the total # of people living on one side of the river and double it to find the total.

b) Area of ith slice = 6 miles $\times \, dx$

c) Population in ith slice = $\rho(x_i)\,(6\,dx)$

d)

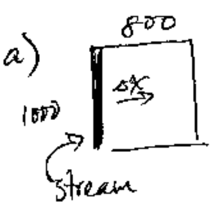
$$2\left[\sum_{i=0}^{n} \rho(x_i)\cdot 6\,\Delta x\right]$$

e) $2\int_0^2 \rho(x)\cdot 6\,dx = 12\int_0^2 (10{,}000 - 800x)\,dx$

$= \cancel{12\int_0^2} = 12\left[10{,}000x - 400x^2\right]_0^2$

$= 12\,(20{,}000 - 1600) = 12(18{,}400) = \boxed{220{,}800 \text{ people}}$

(#4) a)

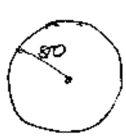


$\int_0^{800} f(x)\cdot 1000\,dx = 1000\int_0^{800} 50 - 0.3\sqrt{x}\,dx$

$= 1000\left[50x - (0.3)\left(\frac{2}{3}x^{3/2}\right)\right]_0^{800}$

$= 1000\left[50(800) - (0.3)\left(\frac{2}{3}\right)(800)^{3/2}\right] = \boxed{35{,}474{,}517 \text{ ears}}$

b)



$\int_0^{80} f(x)\cdot 2\pi x\,dx = 2\pi\int_0^{80} x(50 - 0.3\sqrt{x})\,dx$

$= 2\pi\int_0^{80} 50x - 0.3x^{3/2}\,dx = 2\pi\left[25x^2 - (0.3)\left(\frac{2}{5}x^{5/2}\right)\right]_0^{80}$

$= 2\pi\left[25(80)^2 - (0.3)\left(\frac{2}{5}\right)(80)^{5/2}\right] = \boxed{962149 \text{ ears}}$

(#5) area of box bottom = 5×16

$$\int_0^{25} \rho(h)\,\underbrace{(5\times 16)\,dh}_{\text{volume of slice}} = \int_0^{25} \frac{4}{h+10}(80)\,dh = 320\int_0^{25}\frac{1}{h+10}\,dh$$

$$= 320 \ln|h+10| \Big|_0^{25} = 320\left[\ln 35 - \ln 10\right]$$

$$= \boxed{401 \text{ raisins}}$$

(#6) radius = 10 inches

$$\text{holes} = \int_0^{10} \rho(r)\,2\pi r\,dr = 2\cancel{\pi}\int_0^{10}\frac{1010}{\cancel{\pi}(r^2+1)^2}\,r\,dr$$

$$= 2020\int_0^{10}\frac{r}{(r^2+1)^2}\,dr \qquad u = r^2+1,\quad du = 2r\,dr$$

$$= \frac{2020}{2}\int_0^{10=r}\frac{du}{u^2} = 1010\cdot\left[-u^{-1}\right]_{r=0}^{r=10}$$

$$= -1010\left[\frac{1}{r^2+1}\right]_0^{10}$$

$$= -1010\left[\frac{1}{101} - \frac{1}{1}\right] = \boxed{1000 \text{ holes}}$$

(#9) a) spherical "shells" (hollow)

b) $4\pi r^2\,dr$ = volume of infinitesimal shell

c) geometrically: the volume of a shell is the surface area of a "flat" shell, extended outward a distance dr

numerically: $4\pi r^2 dr$

(#12) $$\int_0^{8000}\rho(r)\,4\pi r^2\,dr = \int_0^{8000}\left(\frac{40{,}000}{1+0.0001r^3}\right)4\pi r^2\,dr = 160{,}000\pi\int_0^{8000}\frac{r^2\,dr}{1+0.0001r^3}$$

$u = 1+0.0001r^3$, $du = 0.0003r^2\,dr$

So, use $$\frac{160{,}000\pi}{0.0003}\int_0^{r=8000}\frac{du}{u} = \frac{160{,}000\pi}{0.0003}\ln\left|1+0.0001r^3\right|\Big|_0^{8000}$$

$$= (1.68\times 10^9)\left[\ln(1+0.0001(8000)^3)\right] = \boxed{2.97\times 10^{10}\text{ kg}}$$